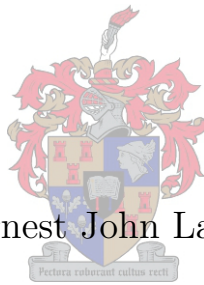


Decision support with respect to facility location and fleet composition for FoodBank Cape Town



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Declaration

By submitting this thesis electronically, I declare that the entirety of the work contained therein is my own, original work, that I am the owner of the copyright thereof (unless to the extent explicitly otherwise stated) and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

March, 2013

Abstract

FoodBank South Africa is a non-profit organisation formed to establish a national network of community foodbanks in urban and rural areas of South Africa, with all participants working towards the common goal of eliminating hunger and food insecurity. *FoodBank Cape Town* was the first of these community foodbanks launched in South Africa on 2 March 2009. The operations of *FoodBank Cape Town* include sourcing food and redistributing it to agencies (social services organisations running feeding programmes). Currently the majority of the food is sourced from the retail sector and then redistributed to approximately two hundred agencies.

The logistics involved in both sourcing and distributing food are vital to the efficient functioning of *FoodBank Cape Town*. Since the costs associated with these logistics operations are very high, streamlining these operations has been identified as a priority area for efficiency improvement. The focus in this thesis is on the distribution logistics involved, specifically focussing on a facility location problem according to which *FoodBank Cape Town* can establish local distribution depots to which it delivers food and from which the agencies collect food assigned to them.

A mixed-integer programming model is formulated for the above facility location problem and small test instances of the problem are solved using different exact and approximate solution methods in order to identify a suitable solution methodology for the full (large-scale) *FoodBank Cape Town* facility location problem. The full facility location problem is solved approximately by means of a meta-heuristic solution method in the more highly constrained instances, while an exact method is selected for solving the lesser constrained instances. The problem is first solved based on the distances between the warehouse and the depots as well as the distances between the agencies and the depots, for the twenty four instances where 17 to 40 depots are located. The model is then developed further to incorporate the cost of distribution. This cost-based facility location model is solved with a view to minimise the cost of food distribution from the warehouse to the depots and the cost of food distribution incurred by each agency to collect food from its assigned depot. A basic vehicle routing technique is applied to the cost-based facility location solution and the associated costs of the distribution are updated. This cost-based solution updating process is performed iteratively until the solution converges. Since the cost of food distribution depends on the vehicle fleet composition used, a vehicle fleet composition comparison of possible *FoodBank Cape Town* vehicles is performed to determine the most desirable vehicle fleet composition to be used for the distribution of food to depots.

The results of the *FoodBank Cape Town* facility location problem and vehicle fleet composition comparison are presented and recommendations are made to *FoodBank Cape Town* regarding the preferred number of depots, the location of these depots and the preferred vehicle fleet composition.

Uittreksel

FoodBank South Africa is 'n nie-winsgewende organisasie wat ten doel het om 'n nasionale netwerk van gemeenskapsvoedselbanke in stedelike en landelike gebiede van Suid-Afrika op die been te bring, waarin al die deelnemers die gemeenskaplike doel nastreef om honger en voedselonsekerheid te elimineer. *Foodbank Cape Town* was die eerste van hierdie gemeenskapsvoedselbanke in Suid-Afrika en is op 2 Maart 2009 gestig. Die take van *Foodbank Cape Town* sluit in die versameling van voedsel en die verspreiding daarvan aan agentskappe (gemeenskaps-organisasies wat voedingsprogramme bestuur). Die oorgrote meerderheid voedsel is tans uit die kleinhandelsektor afkomstig en word aan ongeveer tweehonderd agentskappe versprei.

Die logistiek wat met hierdie versamelings- en verspreidingsprosesse gepaard gaan, is sentraal tot die doeltreffende funksionering van *FoodBank Cape Town*. Aangesien die kostes verbonde aan hierdie logistieke prosesse baie hoog is, is hierdie aktiwiteite as 'n prioriteitsarea vir verbetering geïdentifiseer. Die fokus in hierdie tesis val op die logistiek verbonde aan die verspreiding van voedsel deur *FoodBank Cape Town*, en meer spesifiek op die probleem van die plasing van 'n aantal lokale verspreidingsdepots waar *FoodBank Cape Town* voedsel kan aflewer en waar die agentskappe dan voedsel wat aan hulle toegeken is, kan gaan afhaal.

'n Gemengde heeltallige-programmeringsmodel word vir die bogenoemde plasingsprobleem geformuleer en klein gevalle van die model word deur middel van beide eksakte en benadere oplossings-tegnieke opgelos om sodoende 'n geskikte oplossingsmetode vir die volle (grootskaalse) *FoodBank Cape Town* plasingsmodel te identifiseer. Die volle plasingsmodel word aan die hand van 'n metaheuristiese oplossingstegniek benaderd opgelos vir hoogsbeperkte gevalle van die model, terwyl minder beperkte gevalle van die model eksak opgelos word. Die plasingsmodel word eers met die oog op die minimering van afstande tussen die pakhuis en verspreidingsdepots sowel as tussen die verspreidingsdepots en agentskappe vir die vier-en-twintig gevalle van die plasing van 17 tot 40 verspreidingsdepots opgelos. Die model word dan verder ontwikkel om ook die koste van die verspreiding van voedsel in ag te neem. Die koste-gebaseerde plasingsmodel word opgelos met die doel om die voedselbankkoste van voedselverspreiding vanaf die pakhuis na die lokale verspreidingsdepots sowel as die agentskapkoste van die afhaal van voedsel vanaf verspreidingsdepots te minimeer. 'n Basiese voertuigroeteringstegniek word op die koste-gebaseerde plasingsmodel toegepas en die verspreidingskoste word dienooreenkomstig aangepas. Hierdie aanpassingsproses van die koste-gebaseerde oplossing word herhaal totdat die oplossing konvergeer. Aangesien die koste van voedselverspreiding afhang van die voertuigvlootsamestelling, word 'n vergelyking tussen moontlike vlootsamestellings vir *FoodBank Cape Town* getref om die mees geskikte samestelling van voertuie vir die verspreiding van voedsel te vind.

Die resultate van die *FoodBank Cape Town* verspreidingsdepot-plasingsprobleem en vlootsamestellingsvergeelyking word aangebied en 'n aanbeveling word aan *FoodBank Cape Town* gemaak in terme van 'n geskikte aantal verspreidingsdepots, waar hierdie depots geleë behoort te wees, en 'n geskikte voertuigvlootsamestelling vir die verspreiding van voedsel.

Terms of Reference

During 2009 the author had the opportunity to attend the annual conference of the Operations Research Society of South Africa (ORSSA) in Stellenbosch. At that stage the author was in the final stages of an honours degree in Operations Research and considering pursuing further studies. During the conference the author met with Mr Timothy Blake, a masters student in Operations Research at the University of Cape Town, his supervisor, Prof Theo Stewart and his co-supervisor, Dr Esbeth van Dyk. Mr Blake presented work from his masters thesis at the conference, entitled *Aiding decision making for FoodBank Cape Town*. He had done work for the organisation FoodBank Cape Town, focussing on the location of a warehouse and the allocation of food to agencies they served. FoodBank Cape Town had only started operations in 2009, but Mr Blake had been involved even before the actual commencement of operations. During his presentation he mentioned a few areas that he thought could be the focus of future research projects. Some further communication with Mr Blake and Dr van Dyk ensued towards the end of 2009 and again at the beginning of 2010.

Dr van Dyk scheduled a meeting with various members of FoodBank Cape Town and FoodBank South Africa staff during the first week of February 2010. At this meeting the history behind FoodBank Cape Town and FoodBank South Africa was discussed, and the vision of the organisation was shared with the author. Ideas on the development of the organisation were discussed, leading to a few focus areas as potential topics for this thesis being highlighted. A visit to the FoodBank Cape Town warehouse and further meetings with members of FoodBank Cape Town staff involved in sourcing of food, distribution of food and agency relations followed.

Early in 2010 a further meeting was held with Prof Stewart, Dr van Dyk and Prof Jan van Vuuren. Prof Stewart was supervising Mr Neil Watson, a masters student of his at the time, who was also planning on focussing work for his masters thesis on a suitable FoodBank Cape Town topic. During this meeting it was decided that Mr Watson would focus his masters work on a food allocation model and that the author's masters thesis focus would be on distribution logistics. Towards the end of 2010 a meeting was held at the FoodBank South Africa offices with members of their staff, Prof van Vuuren, Dr van Dyk, Prof Stewart, Mr Watson and Dr Leanne Scott, a co-supervisor of Mr Watson. During this meeting ideas were shared on both project topics and an alignment of data between the two projects was performed.

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Table of Contents

List of Figures	xv
List of Tables	xvii
List of Algorithms	xix
List of Acronyms	xxi
1 Introduction	1
1.1 What is foodbanking?	2
1.1.1 The role of a foodbank	2
1.1.2 How the notion of foodbanking was established	3
1.1.3 The Global Foodbanking Network	3
1.2 FoodBank South Africa	4
1.3 The scope and objectives of this thesis	5
1.3.1 The scope	5
1.3.2 The objectives	6
1.4 Thesis organisation	8
2 Preliminary Concepts	9
2.1 Facility Location	10
2.1.1 K -Centre Problem	10
2.1.2 K -Median Problem	11
2.1.3 The Fixed Charge Location Problem	11
2.2 Solving Linear and Integer Programming Models	12
2.2.1 The simplex and dual simplex algorithms	12
2.2.2 The Branch-and-bound Method	14
2.2.3 Lingo 11.0 Mathematical Solver	15

2.2.4	The Method of Tabu Search	15
2.2.5	Adaptive Tabu Search for the CCP	16
2.3	Vehicle Routing	21
2.3.1	Problem Description	21
2.3.2	The Clarke-Wright Algorithm	22
2.4	Geographic Information Systems	24
2.4.1	Coordinate Systems	24
2.4.2	Data Models	24
2.4.3	ArcGIS	25
2.4.4	Flowmap	25
2.5	Fitting Data to a Theoretical Statistical Distribution	25
2.5.1	The Kolmogorov-Smirnov Test	25
2.5.2	The Anderson-Darling Test	26
2.5.3	The Chi-Squared Test	26
2.6	Chapter Summary	27
3	Facility Location Problem	29
3.1	Problem Description and Methodology	29
3.1.1	Background Information	30
3.1.2	Food Distribution Scenarios	32
3.1.3	Geographical Information System Data	34
3.1.4	Infrastructure Location	34
3.1.5	Demand Data	35
3.1.6	Candidate Sites	35
3.1.7	Distance and Cost	39
3.2	Model Formulation	43
3.3	Application to Test Problems	44
3.3.1	Generation of Test Problems	44
3.3.2	Numerical Results	45
3.4	Application on FBCT Data and Results	50
3.4.1	Application on FBCT Data	50
3.4.2	Results	52
3.5	Chapter Summary	58

4	Facility Location with Vehicle Fleet Composition	61
4.1	Model Formulation	61
4.2	Vehicle Cost Calculation	62
4.2.1	FBCT Vehicle Costs	64
4.2.2	Agency Vehicle Costs	65
4.3	Application of Vehicle Routing	65
4.4	Initial Results	69
4.5	Feedback Process and Final Results	71
4.6	Chapter Summary	76
5	Conclusion	77
5.1	Thesis Summary	77
5.2	Recommendations to FBCT	78
5.3	Feedback from FBCT	79
5.4	Challenges of this Study	79
5.5	Future Work	80
A	Additional Tables and Results	85
A.1	5-ton and 1.5-ton VFC Results	85
A.2	30-depot Cost-based FLP Solution	85
B	Contents of the accompanying compact disk	91

List of Figures

1.1	Simplified view of the foodbanking process	2
3.1	Distribution scenario 1	32
3.2	Distribution scenario 2	33
3.3	Distribution scenario 3	33
3.4	Warehouse and agencies	36
3.5	Warehouse, agencies and assigned hexagons	37
3.6	Warehouse and agency hexagons	38
3.7	National land use for the City of Cape Town	40
3.8	Suitable candidate location hexagons	41
3.9	Central candidate location hexagons	42
3.10	The weekly total distance, total WD distance and total AD distance	51
3.11	The 27 depots selected for the distance-based FLP	56
3.12	Assignment of agencies to depots for the distance-based FLP for 27 depots . . .	57
4.1	The initial results for the total weekly WD cost for 17 to 40 depots	70
4.2	The results for the total cost for four VFCs	72
4.3	Breakdown of the total cost for the VFC of one 5-ton and one 1.5-ton vehicle . .	73
4.4	Breakdown of the total distance for the VFC of one 5-ton and one 1.5-ton vehicle	74
4.5	Comparison of depots selected in the 30-depot instance	75

List of Tables

3.1	The profile of demand for FBCT agencies and test size agencies	45
3.2	The results of LINGO 11.0 and the CCP-ATS for test problems of size 25	46
3.3	The results of LINGO 11.0 and the CCP-ATS for test problems of size 50	47
3.4	The results of LINGO 11.0 and the CCP-ATS for test problems of size 100	48
3.5	The results of the three goodness-of-fit tests	49
3.6	Statistics of the results for problems of size 50	52
3.7	Statistics of the results for problems of size 100	52
3.8	The list of agency hexagon numbers and their corresponding demands	53
3.9	The results (part 1) of the FBCT FLP based on distance for 17 to 40 depots . .	54
3.10	The agency-depot assignment for the 27-depot instance	55
3.11	The results (part 2) of the FBCT FLP based on distance for 17 to 40 depots . .	58
4.1	Fixed costs of vehicles of different load capacities	63
4.2	Variable costs of vehicles of different load capacities	63
4.3	FBCT vehicle costs for three different vehicle load capacities	64
4.4	Agency vehicle costs for three different vehicle load capacities	65
4.5	The travel distances and travel times for four 1.5-ton FBCT vehicles	67
4.6	The vehicle routing for the distance-based FLP with 27 depots	68
A.1	The results (part 1) of the FBCT FLP based on cost for 17 to 40 depots	86
A.2	The results (part 2) of the FBCT FLP based on cost for 17 to 40 depots	87
A.3	The agency-depot assignment for the 30-depot instance	88
A.4	The vehicle routings for the cost-based FLP with 30 depots	89

List of Algorithms

2.1	Initial Solution Algorithm	17
2.2	Procedure negativeDelta	18
2.3	Procedure evaluateDelta	18
2.4	Adaptive Tabu Search Algorithm	19
2.5	Clarke-Wright Algorithm	23
4.1	Trip Assignment	67

List of Acronyms

AA	Auto-mobile Association
AD	Agency-Depot
ATS	Adaptive Tabu Search
CCP	Capacitated Clustering Problem
DC	Distributon Centre
FBCT	FoodBank Cape Town
FBSA	FoodBank South Africa
FEBA	European Federation of Food Banks (Fédération Européenne des Banques Alimentaires)
FLP	Facility Location Problem
GFN	Global Foodbanking Network
GIS	Geographical Information System
GPS	Global Positioning System
LP	Linear Programming
LDV	Light Delivery Vehicle
RFA	Road Freight Association
VCS	Vehicle Cost Schedule
VFC	Vehicle Fleet Composition
WD	Warehouse-Depot

CHAPTER 1

Introduction

Contents

1.1	What is foodbanking?	2
1.1.1	<i>The role of a foodbank</i>	2
1.1.2	<i>How the notion of foodbanking was established</i>	3
1.1.3	<i>The Global Foodbanking Network</i>	3
1.2	FoodBank South Africa	4
1.3	The scope and objectives of this thesis	5
1.3.1	<i>The scope</i>	5
1.3.2	<i>The objectives</i>	6
1.4	Thesis organisation	8

South Africa is one of only a few countries which is able to produce enough food to feed its people. However, nearly 40% of the South African population is food insecure¹ [4], meaning that approximately 19 million people in the country are not able to produce sufficient food for themselves, or do not have accessibility to or the ability to purchase food from the market, at all times.

Food Security is a very pressing need in South Africa which has led to the formation of the South African Forum for Food Security early in 2008, whose mandate it is to address the approach to hunger relief in the country. The lack of food security in South Africa is not a matter of inadequate supply, since the country has the ability to produce enough food. The problem rather lies in the access to the food and the underlying logistics involved [22].

With the help and guidance of the *Global FoodBanking Network* (GFN), the South African Forum for Food Security was successful towards the end of 2008 in convincing a number of leading hunger relief organisations, including *Feedback Food Redistribution*, *Lions Food Project*, *Robin Good Initiative* and *Johannesburg Foodbank*, to agree to amalgamate their operations to form a new organisation, *FoodBank South Africa* (FBSA).

¹Food Security is defined as *access by all households at all times to adequate, safe and nutritious food for a healthy and productive life*.

1.1 What is foodbanking?

Simply stated, foodbanking is a system which captures surplus food in order to provide redistribution agencies with food to feed people in need.

A foodbank is an organisation which sources food from donors on behalf of the redistribution agencies that it supplies. The donors are typically government agencies, farms and fisheries, food manufacturers, food wholesalers, supermarkets and consumers. The agencies to whom the food is issued then provide the food to the needy (beneficiaries). These agencies include feeding schemes, shelters, school feeding programs, soup kitchens, nutritional centres and other non-profit organisations. Large volumes of the food donated would otherwise have gone to waste. Reasons for food donations include labelling errors, food near its expiration date, brand discontinuation, inventory surplus and damaged packaging. A simplified view of the foodbanking process is presented in Figure 1.1.

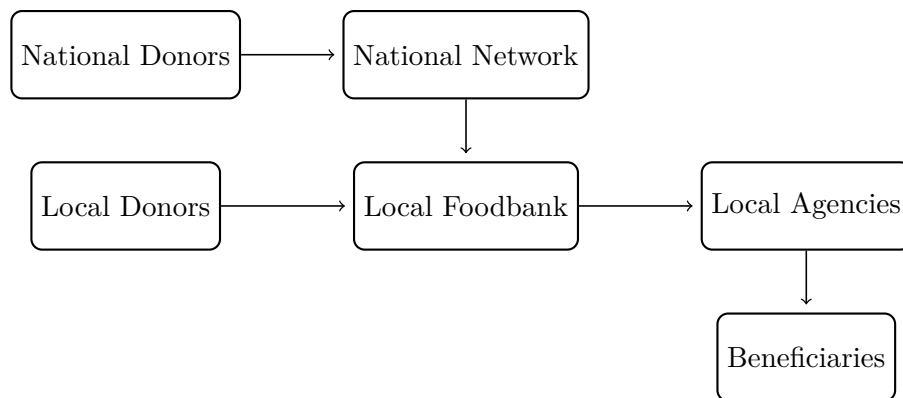


Figure 1.1: A simplified view of the foodbanking process.

1.1.1 The role of a foodbank

As mentioned above, a foodbank is a non-profit organisation which distributes unused food to agencies which, in turn, feed the hungry in the local community. The foodbank must obtain, store and transport food in a safe and coordinated way to serve the community effectively.

A foodbank typically obtains the majority of the food it receives through donations. These donations originate from various sources including retailers, manufacturers and producers. Food may also be bought to supplement the donated food. By acting on behalf of various agencies a foodbank is typically able to achieve economies of scale and obtain purchased food at cheaper prices than the agencies can by themselves. This requires foodbanks to cultivate sourcing and distribution professionalism which allows the distribution of increased food volumes at lower costs.

The procurement of food is the starting point of the foodbanking process. Some donors may deliver the food to the foodbank warehouse, but the majority of food must be collected by the foodbank itself. Once the food has reached the warehouse, branding labels have to be removed after which the food is sorted and stored. The food is then either delivered to or collected by agencies (on a daily or weekly basis, depending on the arrangement between the agency and the foodbank). It is also important to the foodbank to arrange food into nutritionally sensible packages before it is distributed to the agencies. Some foodbanks may have programs

that deliver food directly to the community, but the fundamental purpose of a foodbank is to facilitate adequate flow of food to the hungry through agencies [3].

A foodbank finally also serves as a vehicle to generate greater public awareness and involvement in the fight to end hunger [57]. It is important to engage all sectors of society in the process in order to fight hunger collectively.

1.1.2 How the notion of foodbanking was established

The first foodbank (St. Mary's Food Bank) was established by John van Hengel in Phoenix, Arizona in the United States of America. When van Hengel was collecting food donations while volunteering at St. Vincent de Paul², he learned that grocery stores disposed of food with damaged packaging and food that was near its expiry date. Acting upon this information he organised that this food be donated to the St. Vincent de Paul dining room. Soon St. Vincent de Paul was receiving more donated food than the organisation's one dining room needed. This led to the development of the food "banking" concept, where companies and individuals could make a donation or "deposit" of food or funds and then agencies could make "withdrawals."

Van Hengel approached the St. Mary Basilica with the food "banking" idea and proposed using St. Mary's as a central location where agencies could collect food, at no cost, to feed the needy they served. This resulted in the founding of the St. Mary's Food Bank in 1967. During the first year thirty six agencies benefited from the more than 250 000 pounds of food collected. The success of the operation led to the notion of foodbanking spreading to cities across the United States of America. Van Hengel was greatly assisted by businessman Alan Merrett, who had considerable experience in the food industry. Merrett was responsible for greatly expanding the resource network [53].

In 1976 van Hengel founded Second Harvest as a consulting organisation to assist those wanting to start foodbanks. Today the organisation is known as Feeding America and is the leading domestic hunger relief charity in the United States of America with over two hundred foodbanks in their national network [19, 53].

The concept of foodbanking continued to spread and in 1981 the first Canadian foodbank, Edmonton's Food Bank, was launched [15]. In July 1984 a foodbank was launched in Paris, France and in 1986 the *European Federation of Food Banks* (FEBA) was founded in order to provide a united organisation to address European institutions and multinational companies [17]. With the assistance of FEBA foodbanking operations have grown in Europe; there are currently over two hundred foodbanks in twenty one different European countries. Van Hengel continued an active role in foodbanking, establishing the organisation Food Banking Services Inc to consult to foodbanks in various countries around the world.

1.1.3 The Global Foodbanking Network

In 2006 four national foodbanking networks from North and South America established the GFN. The GFN is a non-profit organisation dedicated to creating, supporting and strengthening foodbanks and foodbanking networks around the world.

The GFN currently shares expertise with foodbanks in nineteen countries in order to develop and strengthen foodbanks in these countries. It also provides a support structure to the local

²St. Vincent de Paul Society is an international Catholic volunteer organisation dedicated to serving the poor and providing personal help to people in any kind of need [51, 52].

leadership running foodbanking operations in the various countries. The GFN is currently evaluating the potential for more foodbanking networks in the Middle East, Latin America, Asia and Africa, while working collaboratively with the FEBA to alleviate hunger through foodbanking by sharing best practices, mobilising resources and promoting hunger relief efforts worldwide.

Moreover, the aims of the GFN are to share foodbanking best practises in order to increase the impact of foodbanking by delivering more food to more people, to build relationships with the global food industry, and to expand the reach of foodbanking operations so as to improve the effectiveness of foodbanking networks around the world. One of the means of achieving these aims is the founding of the Food Bank Leadership Institute, where GFN staff, food industry representatives and food security experts provide training and share knowledge with respect to factors critical to developing and enhancing the effectiveness of foodbanks globally.

1.2 FoodBank South Africa

FBSA was established in 2008 by the South African Forum for Food Security under the guidance of the GFN. As mentioned above, several hunger relief organisations amalgamated their operations to form one organisation in order to fight hunger more effectively in South Africa.

FBSA is working to establish a nationwide network of community foodbanks in urban and rural areas of the country, with all participants working towards the common goal of eliminating hunger and food insecurity. The first community foodbank was launched on 2 March 2009, when *FoodBank Cape Town* (FBCT) was launched. A further three foodbanks, FoodBank Johannesburg, FoodBank Durban and FoodBank Port Elizabeth, were established during 2009 and the national network of community foodbanks was under way. These four urban foodbanks not only directly serve their respective cities, but also have a responsibility towards the broader area they represent in terms of nurturing satellite³ foodbanks. A further fifteen locations have been identified as possible locations for a foodbanks in the future.

During the first year of operation of FBSA, more than 6 000 tonnes of food and essential non-food groceries, valued at 76 million Rands, were distributed to the agencies served. In 2010 FBSA was supplying food to almost 1 100 agencies country wide, resulting in an estimated 203 000 beneficiaries throughout South Africa. These agencies serve three target areas: Child and Youth Development⁴, Adult Development⁵ and Social Welfare⁶.

FBSA as a national body is able to serve local foodbanks by engaging with donors on a national level as well as harnessing support from national and local government. FBSA also runs various fund-raising initiatives and events, such as Fill the Gap Club⁷, and in so doing not only raises funds, but also raises awareness of the fight against hunger.

Since FBCT was the first foodbank to be established in South Africa, it is considered to be an example that the other foodbanks in the country are following. FBCT operates from its

³A small-scale foodbank operation in a nearby city or town.

⁴Child and Youth Development accounts for approximately 42% of agencies, including pre-schools, shelters for orphans and vulnerable children, and school feeding schemes.

⁵Adult Development accounts for approximately 40% of agencies, including soup kitchens and nutritional feeding centres.

⁶Social Welfare, accounts for approximately 18% of agencies, including aged care, disabled care and care for the terminally ill.

⁷An initiative providing individuals, families or companies with the opportunity to donate funds monthly to help end hunger and malnutrition in South Africa.

warehouse in Philippi East, a location selected as a result of decision support in the form of a masters thesis in operations research [3]. The majority of food is sourced from the retail sector and *distribution centres* (DCs). There are currently ± 200 agencies to which it supplies food. FBCT delivers most of the food to the agencies or depots assigned to the agencies, but a minority collect food directly from the FBCT warehouse. A decision support system for the allocation of food resources was developed in 2011 [57], but it had not yet been implemented at the time of writing this thesis. FBCT currently provides approximately 12 million meals annually.

1.3 The scope and objectives of this thesis

The techniques of Operations Research appear to be well suited to assist in improving food-banking operations at FBCT. The range of areas to which these techniques may be applied is, however, very broad. It is therefore important to understand FBCT's needs and its priorities in terms of areas requiring focus before decision support research is undertaken. These needs and priorities have informed the scope of and objectives pursued in this thesis, as described in this section.

1.3.1 The scope

Initial meetings were held with various members of FBSA and FBCT staff [1, 33, 34, 35, 38, 45, 47, 48] at the beginning of 2010. During these meetings the FBSA foodbanking process and objectives were explained to the author in order to allow him to develop an understanding of which areas would be best suited to the development of decision support models. Numerous aspects of FBCT's operations were highlighted and the relationships between the aspects of its operations were discussed. From these initial meetings three major areas were identified to be in need of objective decision support:

- *Food allocation* is both the daily and weekly process of food allocation to agencies. It was seen as a short-term focus area (satisfying current agencies' needs) which may be extended to a long-term focus area (supporting more agencies in the future). A need was identified to further develop FBCT's allocation model, addressing the allocation policies and strategies and the efficiency of the allocation process. Work had already been done on developing the allocation model by Timothy Blake [3] and further development and expansion of the model was required for the food distribution process.
- *Distribution logistics* involves improving the food distribution system and the logistical execution. The cost to FBCT of collection and distribution of food is very high, both in terms of distances travelled by delivery vehicles (impacting on operating costs) and delivery vehicle utilisation (impacting on fixed costs). Assistance in minimising distribution costs was identified as a medium-term focus area once the operations were more established, the focus being mainly on depot creation and distribution fleet composition (including vehicle routing).
- *Activity-based costing* is the process of determining the cost that each operation area contributes to the overall cost of a meal. This focus area is centred on evaluating the FBCT operations and how the costs associated with each area within their operations affect the cost of "producing" a meal. This process also includes an evaluation of plans for

future operations and the result of these future operations on the activity-based costing. It was identified as a short-term focus area that would grow with long-term planning.

Later during 2010 a fourth area was identified:

- *Future foodbank locations.* This focus area centres on developing a poverty map of South Africa and using it to determine ideal locations for new foodbanks yet to be established. This process should take into account population levels, poverty/need, accessibility and cost.

The *food allocation* and the *distribution logistics* focus areas were seen by FBCT as top priority areas. The *future foodbank locations* was viewed as significant, but more of a long-term focus area. The focus area of *activity-based costing* was seen as important, but the outcomes were initially more focussed on immediate assessment and then a long-term continuation. It also appeared to require more project management skills than mathematical modelling skills and techniques. The *food allocation* focus area was adopted as a thesis topic by Neil Watson [57], an Operations Research student at the University of Cape Town. This resulted in the focus for this thesis being on the *distribution logistics*. Furthermore, due to the nature of the *distribution logistics* focus area, the timing of the research required appeared to be well aligned with when FBCT would be making decisions with respect to logistical operations, such as depot selection and vehicle fleet composition.

The logistics involved in both sourcing and distributing food are vital to the efficient functioning of FBCT. Since the costs associated with these logistics operations are very high, streamlining these operations has therefore been identified as a priority area for efficiency improvement. FBCT expressed an intention to establish local distribution depots within the area that they service. These local distribution depots are to provide a location from which agencies allocated to the depots would be able to collect the food earmarked for them. In this way FBCT would not have to deliver food to each agency individually, but rather to the distribution depots only, thereby saving on distribution costs.

Research is carried out in this thesis with a view to assisting FBCT in decisions regarding the location of their local distribution depots, so that transportation costs associated with the distribution of food items to local agencies may be minimised. There is also a possibility that FBCT may phase out its old fleet of distribution vehicles and obtain a new fleet that would be more efficient in carrying out their collection and distribution of food. The research in this thesis further entails decision support with respect to the composition (quantity and specification of capacities) of the fleet of distribution vehicles that FBCT should ideally possess to deliver food to the new distribution depots.

1.3.2 The objectives

As mentioned above, the aim of the work in this thesis is to provide decision support to FBCT with respect to the location of their local distribution depots and the composition of their distribution vehicle fleet. This aim is achieved by modelling FBCT's facility location problem for local distribution depots mathematically and by performing a vehicle fleet composition comparison, implementing these mathematical models programmatically and interpreting the results obtained. The interpretation of the model results should then lead to a set of recommendations to FBCT, providing them with decision support in these areas.

In order to achieve these aims the following objectives are pursued in this thesis:

I To *review* the relevant literature from the sub-disciplines of facility location theory, mathematical programming and simulation.

II Data Collection and Preparation

- (a) To *obtain* data for the FBCT facility location problem and vehicle routing problem and to *prepare* these data further using tools from the field of *geographical information systems* (GIS).
- (b) To *determine* factors influencing FBCT's depot location and to *select* candidate sites suitable for the establishment of local depots.
- (c) To *collect* data pertaining to transportation and depot set-up costs and to *calculate* these costs.

III Facility Location Problem

- (a) To *formulate* a model to solve the facility location problem for FBCT.
- (b) To *generate* and *document* test instances of different small problem sizes for the model formulated.
- (c) To *implement* a meta-heuristic to solve approximately larger or more highly constrained instances of the model relatively quickly and to *compare* heuristic solutions to exact solutions for the test instances in terms of quality of solution and execution time.
- (d) To *solve* the FBCT depot location problem for different numbers of depots, based on distance.
- (e) To *solve* the FBCT depot location problem for different numbers of depots, based on cost.

IV Vehicle Fleet Composition Comparison

- (a) To *apply* the model formulated for facility location in conjunction with different FBCT vehicle fleet compositions, including the application of vehicle routing to the given FBCT vehicle fleet composition.
- (b) To *compare* different vehicle fleet compositions and *determine* critical points where a change in composition would be advisable to FBCT.

V To *use* the results to *assist* FBSA and FBCT in answering various questions related to the distribution of food and the costs involved. These questions are based around the following issues:

- (a) Preferred number of depots,
- (b) Preferred vehicle fleet composition, and
- (c) The breakdown of the distance travelled and cost of distribution between FBCT and the agencies it serves.

1.4 Thesis organisation

This introductory chapter is followed by a chapter containing a review of models in the operations research literature related to the facility location problem of FBCT. The chapter also contains a review of appropriate solution techniques for these models and the software programs used in this thesis.

A facility location model (based on distance) is formulated specifically for FBCT facility location problem in Chapter 3. Small test instances of this model are generated and then solved using an exact linear/integer programming method. It is found that this exact solution method cannot be applied to all facility location problems of the dimensions required by FBCT's agency data due to the computational burden associated with the method. A tabu search meta-heuristic solution for the model is therefore developed and the solutions that it produces are then compared to those of the exact solution method for these small test instances in order to gauge the quality of the heuristic solutions. The tabu search method is finally applied to the full FBCT facility location model, investigating the effects of different numbers of depots. It is found that the tabu search method is suitable for the more highly constrained instances (a small number of depots) while the exact method may still be used to solve less constrained instances (larger numbers of depots).

In Chapter 4 the cost of food distribution is considered in some detail. A breakdown of vehicle costs is provided and these costs are then supplied as the input to the facility location model formulated in Chapter 3, adapted to minimise costs instead of distance. This is achieved by applying a simple vehicle routing algorithm to the facility location solutions obtained by the model of Chapter 3 and calculating the initial costs. Vehicle costs are affected by changes in distances travelled and since vehicle routing changes the distances vehicles are expected to travel, an iterative cost updating is performed and the model formulated in Chapter 4 is solved iteratively to determine whether the facility location problem solution converges as vehicle-related costs are updated. A comparison of different vehicle fleet compositions is also carried out during this process.

In the concluding chapter, Chapter 5, an overview of the work in this thesis is presented. The objectives of the thesis are reviewed to determine whether they were successfully achieved. The decision support assistance offered to FBCT and the feedback obtained from FBCT are also discussed. The difficulties faced during the execution of this work are highlighted and the chapter closes with some ideas with respect to follow-up work.

CHAPTER 2

Preliminary Concepts

Contents

2.1	Facility Location	10
2.1.1	<i>K</i> -Centre Problem	10
2.1.2	<i>K</i> -Median Problem	11
2.1.3	The Fixed Charge Location Problem	11
2.2	Solving Linear and Integer Programming Models	12
2.2.1	The simplex and dual simplex algorithms	12
2.2.2	The Branch-and-bound Method	14
2.2.3	Lingo 11.0 Mathematical Solver	15
2.2.4	The Method of Tabu Search	15
2.2.5	Adaptive Tabu Search for the CCP	16
2.3	Vehicle Routing	21
2.3.1	Problem Description	21
2.3.2	The Clarke-Wright Algorithm	22
2.4	Geographic Information Systems	24
2.4.1	Coordinate Systems	24
2.4.2	Data Models	24
2.4.3	ArcGIS	25
2.4.4	Flowmap	25
2.5	Fitting Data to a Theoretical Statistical Distribution	25
2.5.1	The Kolmogorov-Smirnov Test	25
2.5.2	The Anderson-Darling Test	26
2.5.3	The Chi-Squared Test	26
2.6	Chapter Summary	27

This chapter contains a review of facility location models in the operations research literature and a review of appropriate solution techniques for these models. The chapter also includes a description of software used in this thesis, a description of geographic information systems and an explanation of fitting data to a theoretical statistical distribution.

2.1 Facility Location

Three facility location problems which may be used to place multiple facilities in order to meet demand are reviewed in this section. Each problem can be represented by a graph $G(V, E)$ of order N and size M in which both the edges and vertices are weighted. The set of vertices is denoted by $V = \{v_1, \dots, v_N\}$ and the set of edges is denoted by $E = \{e_1, \dots, e_M\}$. Two additional sets are defined in terms of the vertices of G , namely the set of demand points represented by $I = \{u_1, \dots, u_n\} \subseteq V$, and the set of candidate facility sites represented by $J = \{w_1, \dots, w_m\} \subseteq V$. A vertex $u_i \in I$ has a weighting which represents the demand to be met at that vertex, denoted by h_i , while a vertex $w_j \in J$ has a weighting which represents the capacity, denoted by C_j , of a facility when located at that vertex. The sets I and J are not mutually exclusive; a vertex may be both a demand point and a candidate facility site, only a demand point, only a candidate facility, or neither. The weight of each edge represents the distance or cost incurred by utilising the edge and is denoted by either a distance factor d_{ij} or a cost factor c_{ij} .

2.1.1 K -Centre Problem

The K -centre problem is the problem of locating a set of $1 \leq K \leq m$ facilities in the graph $G(V, E)$ so that the largest demand-weighted distance between a vertex and the facility to which it is assigned is a minimum over all demand vertices and all facilities located. This problem may be formulated as a mixed-integer linear programming problem [9, 39], with the objective of minimising the *largest* demand-weighted distance, that is

$$\text{minimising } g = \max_{i \in I, j \in J} \{h_i d_{ij} y_{ij}\}, \quad (2.1)$$

between a demand point and the facility to which it is assigned, where

$$y_{ij} = \begin{cases} 1 & \text{if the demand at vertex } u_i \text{ is assigned to the facility located at vertex } w_j \\ 0 & \text{otherwise} \end{cases}$$

is a binary decision variable. Upon defining the additional binary decision variable

$$x_j = \begin{cases} 1 & \text{if facility is located at vertex } w_j \\ 0 & \text{otherwise,} \end{cases}$$

the minimisation of g in (2.1) should be carried out subject to the constraints

$$\sum_{j \in J} x_j = K, \quad j = 1, \dots, m, \quad (2.2)$$

$$\sum_{j \in J} y_{ij} = 1, \quad i = 1, \dots, n, \quad (2.3)$$

$$y_{ij} \leq x_j, \quad \begin{aligned} i &= 1, \dots, n, \\ j &= 1, \dots, m, \end{aligned} \quad (2.4)$$

$$x_j, y_{ij} \in \{1, 0\}, \quad \begin{aligned} i &= 1, \dots, n, \\ j &= 1, \dots, m. \end{aligned} \quad (2.5)$$

Constraint set (2.2) ensures that exactly K facilities are located, while constraint set (2.3) ensures that the demand of each demand point is assigned to one facility. Constraint set (2.4) ensures that demand is only assigned to a facility which has actually been located in the network. In the K -centre problem the demand of a demand point is met fully by some appropriate facility located.

2.1.2 K -Median Problem

The K -median problem is the problem of locating a set of $1 \leq K \leq m$ facilities in the graph $G(V, E)$ so as to minimise the *total* demand-weighted distance between the facilities and the demand they service, over all demand points. Hakimi [31] has shown that a solution to the K -median problem can always be found with facilities only placed at the vertices of $G(V, E)$. Therefore the set of candidate sites for facilities may be restricted to the set V without loss of generality.

The K -median problem may be formulated as a mixed-integer linear programming problem [9, 21] with the objective of

$$\text{minimising } g = \sum_{i=1}^n \sum_{j=1}^m h_i y_{ij} d_{ij} \quad (2.6)$$

subject to the same constraints (2.2)–(2.5) of the K -centre problem.

In the K -median problem, as in the K -centre problem, the demand of a demand point is met in full by an appropriate facility located in $G(V, E)$.

2.1.3 The Fixed Charge Location Problem

The *fixed charge location problem* is the problem of locating a set (of unknown cardinality) of facilities in the graph $G(V, E)$ so as to minimise the fixed cost of locating the facilities and the distribution costs incurred by satisfying the demand at demand points in G . The fixed charge location problem relaxes the constraint (2.2) of the K -centre problem, therefore allowing for the determination of a suitable number¹ of facilities to be located. Two models exist for the fixed charge location problem, namely the *uncapacitated model* and the *capacitated model*.

The Uncapacitated Model

The *uncapacitated fixed charge location model* may be formulated as a mixed-integer linear programming problem [9] with the objective of

$$\text{minimising } g = \sum_{j=1}^m f_j x_j + c \sum_{i=1}^n \sum_{j=1}^m h_i y_{ij} d_{ij}, \quad (2.7)$$

where f_j is the fixed cost of locating a facility at vertex w_j , where c is the distribution cost per unit distance of satisfying a unit of demand and where the other symbols have the same meaning as before.

The first term in (2.7) represents the total fixed cost of all the facilities that are located, while the second term represents the cost associated with satisfying the demand at the various demand points from appropriately assigned facilities. The minimisation of g in (2.7) occurs subject to the constraints (2.3)–(2.5).

An example of the *uncapacitated model* of the *fixed charge location problem* is described in an application involving a large Belgian brewery in [27]. In this paper the *uncapacitated fixed charge location model* is used to determine the distribution of products of a Belgian brewery. In particular, it is used to determine how many depots are required and where to locate these

¹An upper bound on this number is the number of candidate facility sites (i.e. m), while a lower bound is the minimum number of facilities required to meet all demand.

depots. The set of candidate sites contains 34 potential depot locations, while the set of demand points represents about 24 000 customers (this was, however, reduced so that the demand points became 650 regions). In solving the problem, constraint (2.2) was also used. The authors solved the problem for all integer values of K in the range $5 \leq K \leq 34$. The best solution was found when locating 17 depots.

The Capacitated Model

The only difference between the *capacitated model* and the *uncapacitated model* of the fixed charge location problem is that in the *capacitated model* each facility that may be located has a limit (capacity) on the demand that it can meet, whereas no such limits (capacities) are present in the *uncapacitated model*. The various facilities may have distinct capacities and the *capacitated model* may be formulated as a mixed-integer linear programming problem [9] with the objective of minimising the same total cost g as in the *uncapacitated model*, while the constraints

$$\sum_{i \in I} h_i y_{ij} \leq C_j x_j, \quad j = 1, \dots, m \quad (2.8)$$

are added to the set of constraints (2.3)–(2.5) in the *uncapacitated model* formulation. Here C_j denotes the capacity of a facility at candidate site w_j if a facility were to be located there and constraint set (2.8) ensures that the demand assigned to a located facility does not exceed the capacity of the facility.

The *capacitated fixed charge location model* differs from the *uncapacitated fixed charge location model*, *K-median problem* and the *K-centre problem* in its objective function and constraints, which results in the situation where demand is not necessarily met by the nearest/closest facility located.

2.2 Solving Linear and Integer Programming Models

This section contains a brief review of the solution methods used in this thesis to solve linear and integer programming models. The simplex algorithm, the simplex dual algorithm and branch-and-bound algorithm used by the LINGO 11.0 solver are first presented, followed by a description of the notion of a tabu search and a description of an adaptive version of the method of tabu search for solving facility location models.

2.2.1 The simplex and dual simplex algorithms

A *linear programming* (LP) problem is a mathematical programming problem requiring the optimisation of a linear objective function subject to a set of linear constraints. The simplex algorithm is a classical mathematical method for solving LP problems. The feasible region of an LP problem is a convex set S of all points that satisfy all the constraints of the LP problem [59] and for an LP problem that has an optimal solution, there is an optimal solution corresponding to an extremal point² of S . The simplex algorithm searches along the extremal points of the feasible region for an optimal solution to the LP problem.

²In a convex set S , a point $P \in S$ is an extremal point if each line segment that lies completely in S and contains the point P has P as an endpoint of the line segment [59].

If all the constraints of an LP problem are equations and all variables are non-negative, then the LP problem is said to be in standard form. In order to solve an LP problem by means of the simplex algorithm it must be converted to standard form. This conversion may be performed as follows:

- A slack variable s_i is added to the i -th less-than-or-equal-to constraint and the sign restriction $s_i \geq 0$ is imposed.
- A slack variable e_i is subtracted from the i -th greater-than-or-equal-to constraint and the sign restriction $e_i \geq 0$ is imposed.
- Each variable x_i that is unrestricted in sign is replaced by $x'_i - x''_i$ in the objective function, as well as each constraint in which it appears and the sign restrictions $x'_i \geq 0$ and $x''_i \geq 0$ are imposed.

Consider a system $A\mathbf{x} = \mathbf{b}$ of m linear equations in n variables, where $n \geq m$. A basic solution to $A\mathbf{x} = \mathbf{b}$ is obtained by setting $n - m$ variables (the non-basic variables) equal to 0 and solving for the values of the remaining m variables (the basic variables). This assumes that setting the $n - m$ variables equal to 0 yields unique values for the remaining m variables or, equivalently, that the columns of A corresponding to the remaining m variables are linearly independent [59]. Any basic solution to $A\mathbf{x} = \mathbf{b}$ in which all variables are non-negative is a basic feasible solution.

The simplex algorithm proceeds as follows [59]:

1. Convert the LP problem to standard form.
2. Obtain a basic feasible solution, if possible.
3. Determine whether the current basic feasible solution is optimal. This is done by testing whether the objective function can be increased by increasing a non-basic variable from its current value of zero while holding the other non-basic variables at zero. A non-basic variable with a negative coefficient in the row 0 will achieve this.
4. If the current basic feasible solution is not optimal, determine which non-basic variable should become a basic variable and which basic variable should become a non-basic variable to find a new basic feasible solution that has a better objective function value. The non-basic variable with the most negative coefficient in the row 0 is chosen, known as the entering variable. How large the entering variable can be made is determined by the so-called *ratio test*. The ratio

$$\frac{\text{right-hand side of row}}{\text{coefficient of entering variable in row}}$$

is computed for each row in which the entering variable has a positive coefficient and the row with the smallest ratio determines the value of the entering variable.

5. Use elementary row operations to find the new basic feasible solution with an improved objective function. Return to step 3.

If the LP problem is not unbounded and cycling does not occur, then the simplex algorithm continues until an optimal solution is found [59].

Associated with an LP problem, called the *primal* problem, is another LP problem, called the *dual* problem [58]. Given a (primal) LP maximisation problem

$$\begin{array}{ll} \text{maximise} & \mathbf{c}^T \mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}, \end{array}$$

the dual is given by

$$\begin{array}{ll} \text{minimise} & \mathbf{y}^T \mathbf{b} \\ \text{subject to} & A^T \mathbf{y} \geq \mathbf{c} \\ & \mathbf{y} \geq \mathbf{0}. \end{array}$$

The dual simplex algorithm proceeds as follows (for a maximisation problem) [58]:

1. If the right-hand side of each constraint is non-negative, an optimal solution has been found, else at least one constraint has a negative right-hand side.
2. The most negative basic variable is chosen to leave the basis. The pivot row is the row in which this variable is basic. The variable that enters the basis is selected by calculating the ratio

$$\frac{\text{coefficient of } x_j \text{ in row 0}}{\text{coefficient of } x_j \text{ in the pivot row}}$$

for each variable x_j having a negative coefficient in the pivot row and choosing the smallest ratio (absolute value) as the value of the entering variable. Elementary row operations are used to render the entering variable basic in the pivot row.

3. The constraints are tested for infeasibility. If no infeasibility is found, the process returns to step 1. The LP problem is infeasible if any constraint has a negative right-hand side and each variable has a non-negative coefficient.

If $\bar{\mathbf{x}}$ is a feasible solution to the primal LP problem, $\bar{\mathbf{y}}$ a feasible solution to the dual LP problem and $\mathbf{c}^T \bar{\mathbf{x}} = \bar{\mathbf{y}}^T \mathbf{b}$, then $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are the optimal solutions to the primal and dual LPs, respectively [58].

2.2.2 The Branch-and-bound Method

The branch-and-bound method consists of two procedures, branching and bounding, to solve combinatorial optimisation problems and was first presented by Land and Doig [13] in 1960.

Consider a minimisation problem with an objective function $f(\mathbf{x})$ where the decision variable \mathbf{x} ranges over the set S of candidate solutions known as the feasible region. Given a subset $S' \subseteq S$ of candidate solutions, the *branching procedure* returns two or more smaller sets S'_1, S'_2, \dots whose union covers S' . The minimum value of $f(\mathbf{x})$ over S' is then attained at $\min\{\mathbf{x}_1, \mathbf{x}_2, \dots\}$, where $f(\mathbf{x})$ attains a minimum at \mathbf{x}_i within S'_i . The process of recursive application of this procedure may be represented graphically by means of a tree-like data structure, whose nodes represent the subsets S'_1, S'_2, \dots of S' . The *bounding procedure* calculates upper and lower bounds on the minimum value of $f(\mathbf{x})$ within the subset $S' \subseteq S$.

The key idea of the branch-and-bound method is that if a lower bound for a node S_i , a subset of candidate solutions, is greater than an upper bound for another node S_j , then S_i may be

discarded from the search. This procedure is known as *pruning* and is typically implemented by recording the smallest upper bound found in the solution space already examined. The branch-and-bound method terminates when the current candidate set S has been reduced to a single element or the lower bound on the objective function is equal to its upper bound. In both cases the minimum value for $f(\mathbf{x})$ is found within the feasible region.

2.2.3 Lingo 11.0 Mathematical Solver

LINGO 11.0 [41] is an optimisation modelling software package that may be used to solve a variety of mathematical programming problems and incorporates a number of solvers. LINGO 11.0 is used in this thesis to solve mixed-integer linear programming problems.

LINGO 11.0 determines what solver(s) to use by examining a mathematical programming model's structure and mathematical content. If a linear model is detected, LINGO 11.0 uses its linear solver, which features advanced implementations of the primal and dual simplex algorithms (described in §2.2.1). If a non-linear model is detected, LINGO 11.0 uses a non-linear solver. If LINGO 11.0 detects integer restrictions, a branch-and-bound manager enforces these restrictions. Depending upon the nature of the model, the branch-and-bound manager calls either the linear or non-linear solver to solve a subproblem. The branch-and-bound manager also employs preprocessing strategies, heuristics and constraint cut generation routines that may reduce solution time significantly [41].

2.2.4 The Method of Tabu Search

Tabu search is a meta-heuristic technique used for solving combinatorial optimisation problems. Tabu search was first proposed by Fred Glover and the majority of the tabu search principles are attributed to him [14, 28].

A tabu search starts with a typically random initial solution or starting current solution. From the current solution the tabu search moves to an adjacent solution in the solution space. If selected, the adjacent solution then becomes the current solution. The move to an adjacent solution is restricted by certain problem constraints and tabu restrictions.

Tabu searches operate by identifying entire neighbourhoods of solutions which lie adjacent to the current solution. Let S be the finite set of all feasible solutions to a problem. Then a current solution $s \in S$ has a neighbourhood $\mathbf{N}(s) \subseteq S$ in which each solution $s' \in \mathbf{N}(s)$ may be reached from the current solution by performing a single *move* [14]. A move is achieved by a modification applied to the current solution. A candidate list of moves within the neighbourhood of a solution is usually generated using a subset of all possible moves. A value is associated with each candidate solution, representing the change that would occur in the objective function should the corresponding move be made. The best (largest improvement of or smallest non-improvement of the objective function value) admissible (as allowed by restrictions and constraints) candidate solution is selected and the move is performed. The solution obtained by performing this move then becomes the new current solution.

A record is maintained of the best solution that has been encountered as a current solution. Once a move has been performed, the reverse of this move is added to a so-called *tabu list*. The tabu list *tenure* is the number of iterations that moves in the tabu list are not allowed to be performed during the search. This tenure may be a fixed number or a random number between an upper and lower limit. Moves in the tabu list can only be performed if they satisfy an *aspiration*

criterion. The aspiration criterion that is most commonly used is that of allowing a move in the tabu list to be performed if the move improves on the currently best solution encountered previously during the search. The tabu list is therefore a vehicle by which a recency-based memory structure is imposed on the search [11, 14].

During the tabu search long-term and short-term memory structures may also be used to guide the search. This serves the purpose of intensifying and diversifying the search. *Intensification* strategies aim to seek solutions with similar characteristics to the better solutions found during the search. These strategies limit the role of long-term memory in order to try and improve on solutions in the region of the current solution. *Diversification* strategies, on the other hand, aim to avoid leaving large regions of the solution space unexplored and allow long-term memory to have a more dominating role. The simplest diversification strategy used is to perform random restarts of the search. Another restart strategy used is to replace components in the current or best known solution by rarely used components and then restarting the search. A continuous strategy that penalises frequently visited components may also be used. This strategy guarantees exploration of unvisited regions of the solution space by driving the solution from the current region when the penalty is large enough. Penalties are also used when feasible solutions have to satisfy a set of constraints, by relaxing the constraints and penalising the violation of the constraints. This may also be regarded as a diversification of the search as it allows the search to escape from the region of the current solution by violating a constraint and continuing the search in a different region of the solution space. In order to again obtain feasibility, the penalty for the violation may be increased to encourage the search to move away from infeasible solutions. Intensification and diversification strategies rely on the frequency-based memory structure of the tabu search [14, 29].

The tabu search continues until some stopping criterion is met. The stopping criterion may be a fixed number of iterations or a fixed number of iterations without an improvement in the objective function [28].

2.2.5 Adaptive Tabu Search for the CCP

The *capacitated clustering problem* (CCP) is a special case of the capacitated facility location problem with a single source constraint, similar to a *capacitated K -median problem*. Given a set of customers with corresponding demands, it is required in the CCP to partition the customers into a set of K clusters, each with limited capacity. Each customer is to be assigned to only one cluster. For each cluster a median (customer) is found from whom the sum of the distances to all other customers in the cluster is minimised. The objective function is to minimise the sum of the distances from all customers to their assigned medians. The CCP is a hard combinatorial optimisation problem; in fact, the decision problem associated with cluster membership is NP-complete [5]. A metaheuristic approach is therefore required to find good solutions to problem instances that are too large to solve exactly within a realistic time frame. An *adaptive tabu search* (ATS) method is used in this thesis to find good solutions to instances of a problem similar to the CCP. The method uses simple neighbourhood structures and automatically integrates diversification and intensification phases of the search [26].

Before the ATS can be executed a feasible initial solution is required. The initial solution is found by means of an algorithm (see Algorithm 2.1) whose working comprises three stages: *find*, *assign* and *re-compute* [5]. The *find stage* finds K customers to take as approximate medians which achieve a good spread within the set of customers. The two customers who are furthest apart are the first two customers selected as approximate medians. The next approximate median is

selected to maximise the product of the distances from the already selected approximate medians to the new approximate median. This process is continued until K approximate medians have been selected. The *assign stage* assigns all the non approximate median customers to one of the K approximate medians selected during the *find stage*. The customers are ordered using one of four criteria: non-decreasing order of their distance (d_{ij}) to their nearest median, non-decreasing order of their demand-weighted distance (d_{ij}/h_i) to their nearest median, decreasing order of regret function (difference between d_{ij} or d_{ij}/w_i) for the nearest and second nearest approximate median, and decreasing order of demand (h_i) [26]. If capacity allows, the customer is assigned to its nearest approximate median; else it is assigned to the immediately nearest available approximate median, taking the capacity of the approximate medians into account. All customers that are assigned to the same approximate median represent a cluster. In the *re-compute stage* the approximate median of each cluster is re-computed in order to minimise the sum of the distance from each customer in the cluster to the approximate cluster median.

Algorithm 2.1: Initial Solution Algorithm

Data: Set of customers with demand, Cost matrix

Result: Initial set of K medians

- 1 Phase 1 (Find);
 - 2 Find largest $c_{k^*\ell^*}$ value;
 - 3 Set $\zeta_1 \leftarrow k^*, \zeta_2 \leftarrow \ell^*$ and $\phi \leftarrow \{\zeta_1, \zeta_2\}$;
 - 4 Set $i \leftarrow 3$;
 - 5 **while** $i \leq K$ **do**
 - 6 Find next $\zeta_i \in (O - \phi)$, such that $\Pi_{j \in \phi} c_{ij} = \text{Max } \Pi_{j \in \phi, \ell \in (O - \phi)} c_{\ell j}$;
 - 7 Set $\phi \leftarrow \phi \cup \zeta_i$;
 - 8 Set $i \leftarrow i + 1$;
 - 9 Phase 2 (Assign);
 - 10 **for** $k \notin \{\zeta_1, \dots, \zeta_K\}$ **do**
 - 11 Find distance to nearest median;
 - 12 Arrange customers in non-decreasing order of distance.;
 - 13 **if** *Capacity available at nearest median* **then**
 - 14 Assign customer to corresponding median
 - 15 **else**
 - 16 Assign customer to nearest median with available capacity.
 - 17 Re-compute **for** $G_i \leq K$ **do**
 - 18 identify $\zeta_i \in G_i$ such that $Z(G_i) = \sum_{k \in G_i} c_{k\zeta_i} = \text{Min } \sum_{k \in G_i, \ell \in G_i} c_{k\ell}$;
-

Given a current solution s , two neighbourhood generation mechanisms, *pairwise interchange* resulting in the neighbourhood $N_1(s)$ and *shift* or *insertion* resulting in the neighbourhood $N_2(s)$, are considered in the ATS. A *pairwise interchange* is performed by interchanging two customers that are in different clusters. A *shift* is performed by shifting a customer from its current cluster to a different cluster. A search move is performed if one of the neighbours in $N_1(s)$ or $N_2(s)$ result in an improvement of the objective function. Customers are first searched sequentially and systematically for a best *pairwise interchange* and secondly for a best *shift*. Should no improvement be possible, the non-improving move which results in the smallest increase of objective function is performed. Once the move has been performed, the approximate medians of the clusters involved in the move are re-computed (see Algorithm 2.4) to obtain a new current solution.

Procedure 2.2: negativeDelta

```

1 if  $T_1 = 4$  and  $T_2 = 2$  then
2   | Set  $next_{eval} = t + 2h$ ;
3 else
4   | Set  $h \leftarrow$  random integer value between  $[h_{min}, h_{max}]$ ;
5   | Set  $next_{eval} = t + h$ ;
6 if  $\Delta \leq -0.035$  then
7   | Set  $T_1 \leftarrow 4, T_2 \leftarrow 1$ ;
8 else if  $-0.035 < \Delta \leq -0.025$  then
9   | Set  $T_1 \leftarrow 3, T_2 \leftarrow 1$ ;
10 else if  $-0.025 < \Delta \leq -0.015$  then
11   | Set  $T_1 \leftarrow 2, T_2 \leftarrow 0$ ;
12 else if  $-0.015 < \Delta \leq -0.01$  then
13   | Set  $T_1 \leftarrow 1, T_2 \leftarrow 0$ ;

```

Procedure 2.3: evaluateDelta

```

1  $\Delta \leftarrow (previous_{mean} - current_{mean})/previous_{mean}$ ;
2 if  $|\Delta| \leq 0.01$  then
3   | Set  $T_1 \leftarrow 0, T_2 \leftarrow 0$ ;
4   | Set  $h \leftarrow$  random integer value between  $[h_{min}, h_{max}]$ ;
5   | Set  $next_{eval} = t + h/2$ ;
6 else if  $\Delta < -0.01$  then
7   | negativeDelta
8 else if  $\Delta > 0.01$  then
9   | if  $T_1 = 4$  and  $T_2 = 2$  then
10     | Set  $T_1 \leftarrow 3, T_2 \leftarrow 1$ ;
11     | Set  $h \leftarrow$  random integer value between  $[h_{min}, h_{max}]$ ;
12     | Set  $next_{eval} = t + h$ ;

```

The attributes that are chosen to control the search are the inter-cluster edges involved in a move. During a *pairwise interchange* four edges are involved in performing the move. The two edges from the two customers being exchanged to their current cluster's approximate median are deleted and the two edges from the two customers to the other cluster's approximate median are added. A value θ is randomly selected within the tabu tenure range and assigned to each of the edges. The edges are considered tabu-active for the next θ iterations in the sense that a candidate move within θ iterations involving one or more of these edges is subject to restrictions. More specifically, a tolerance parameter T_1 determines the status of a *pairwise interchange* move compromising one or more tabu active edges. T_1 can assume the values 0, 1, 2, 3 or 4 and this value represents the maximum number of tabu-active edges which can be present in a move. If $T_1 = 4$, any candidate move is allowable, while if $T_1 = 0$, only candidate moves with no tabu-active edges are allowed.

During *shift* moves only two edges are involved: the edge from the customer to its current cluster's approximate median is deleted and the edge from the customer to its new cluster's

Algorithm 2.4: Adaptive Tabu Search Algorithm

Data: Set of customers with demand, Cost matrix, Initial set of K medians**Result:** Improved set of K medians

```

1 Set  $t \leftarrow 0$ ;
2 Set  $T_1 \leftarrow 3, T_2 \leftarrow 1$ ;
3 while  $solution_{current} \leq solution_{previous}$  do
4   Set  $solution_{previous} \leftarrow solution_{current}$ ;
5   Perform neighbourhood search and interchange or shift movement;
6   Re-compute Cluster median;
7   Update  $solution_{current}$  store on list;
8 Set  $solution_{best} \leftarrow solution_{current}$ ;
9 Set  $previous_{mean} \leftarrow list_{mean}$ ;
10 Reset list;
11 Set  $h \leftarrow$  random integer value between  $[h_{min}, h_{max}]$ ;
12 Set  $next_{eval} \leftarrow t + h$ ;
13 while stopping criterion not met do
14   for  $t \neq next_{eval}$  do
15     Perform neighbourhood search and interchange or shift movement;
16     Re-compute Cluster median;
17     Update  $solution_{current}$  store on list;
18     if  $solution_{current} < solution_{best}$  then
19        $solution_{best} = solution_{current}$ 
20      $t \leftarrow t + 1$ ;
21    $current_{mean} \leftarrow list_{mean}$ ;
22   if  $solution_{best}$  improved then
23     Set  $T_1 \leftarrow 3, T_2 \leftarrow 1$ ;
24     Set  $h \leftarrow$  random integer value between  $[h_{min}, h_{max}]$ ;
25     Set  $next_{eval} \leftarrow t + h$ ;
26   else
27      $evaluateDelta$ 
28   Set  $previous_{mean} = current_{mean}$ ;
29 Re-compute;
30 for  $G_i \leq p$  do
31   identify  $\zeta_i \in G_i$  such that  $Z(G_i) = \sum_{k \in G_i} c_{k\zeta_i} = Min \sum_{k \in G_i, \ell \in G_i} c_{k\ell}$ ;

```

approximate median is added. Similarly, a value θ is randomly selected within the tabu tenure range and assigned to each of the edges involved in the move, again determining the period for which each edge is tabu-active. For *shift* moves the tolerance parameter T_2 is used. T_2 can only assume the values 0, 1 or 2.

The number of iterations θ for which a move is tabu-active is a random variable uniformly distributed over the interval $[n/4, n/2]$, where n denotes the number of customers. During a search move an aspiration criterion is considered which allows tabu activation rules to be overridden if the move improves on the best objective function value obtained so far.

This approach, proposed in 1996 by Franca and Pureza [26], integrates intensification and diversification strategies. The values T_1 and T_2 are able to adjust automatically for intensification and diversification purposes during the ATS in Algorithm 2.4. Search indicators identify promising regions, resulting in an intensified search of the region, or identify regions with minimal improvements, resulting in a diversification of the search. The level of restrictiveness during the ATS is determined by the values T_1 and T_2 and the tabu tenure. Trajectory patterns are employed to evaluate the behaviour of the objective function over the last h iterations (the length of the horizon to compute and evaluate trajectories). Trajectory patterns are identified by calculating the average of the objective function value during the current search stage and the previous search stage. The average for each stage is taken over h iterations and the relation between the two averages is used to evaluate a trajectory (see Procedure 2.3). The parameter h should be large enough to ensure that improvement possibilities are not overlooked in promising regions and that stagnation is overcome. However, excessively long horizons result in lower quality local optima and delay improvement. A dynamic horizon is considered, using a random integer h within the range $[h_{min}, h_{max}]$. The interval $[h_{min}, h_{max}]$ suggested in [26] corresponds to the interval $[n/10, n/5]$. Three types of trajectory patterns may be observed: *stagnation*, *ascent* and *descent*. *Stagnation* occurs when the averages are approximately the same and requires an increasing restrictiveness level to promote diversification. An *ascent* trajectory occurs when the current average is greater than the previous average in which case tabu restrictions are relaxed to stop diversification. When the current average is less than the previous average a *descent* trajectory is observed in which case mild restrictiveness is established to stimulate a more extensive exploration of the region surrounding the current solution (see Procedure 2.2).

The iteration of the next evaluation is defined in advance by adding $\kappa \times h$ to the current iteration. The parameter κ is a tuning factor. During a high restrictiveness level phase of the search κ is set to a value smaller than 1 so that the length of a diversification phase is limited. Once a descent trajectory is observed κ is set to a value greater than 1 so that the search can benefit from a more thorough search of a promising region of the solution space.

A tightness factor,

$$\tau = \sum_{i \in I} \frac{w_i}{m \cdot W}, \quad (2.9)$$

is employed. This factor is a capacity ratio of the total demand over the total cluster capacities and represents how highly constrained the problem instance is. The range of τ which is acceptable for using the ATS to solve the CCP (approximately) is $[0.82, 0.96]$ [5].

The stopping criterion for the ATS is a pre-specified number of iterations during which no improvement in the best solution was found.

2.3 Vehicle Routing

Problems concerning the distribution of goods from a depot to customers are known as *Vehicle Routing Problems* (VRPs). This section contains a description of the VRP and of a basic VRP heuristic, the Clarke-Wright algorithm.

2.3.1 Problem Description

VRPs are concerned with the distribution of goods to a set of customers within a given time period by a set of vehicles performing the distribution along an appropriate road network. A solution to a VRP determines a set of routes, each followed by a single vehicle (typically starting and ending at a depot) such that all the customers' demands are satisfied, all operational constraints are satisfied and the transportation costs are minimised [55].

The road network along which the distribution of goods occurs may be modelled by means of a graph, in which road sections are represented by edges and in which the depot and customer locations are represented by vertices. Each edge has a cost and travel time associated with it; both are typically related to the length of the road section, but may be influenced by other factors, such as traffic volume. Also, the edges may be directed or undirected, depending on whether traffic flow along the corresponding road section occurs in one direction or in both directions.

Each customer is located at a vertex of the road graph and has a demand which must be satisfied. A time window may be placed by a customer, requiring the customer to be served within the given window. A loading or unloading time is required to deliver goods to or collect goods from a customer. A customer may be restricted to a certain vehicle type servicing it, due to access limitations and loading or unloading requirements. In some cases only partial fulfilment of a customer's demand may be allowed; however, penalties are usually incurred if demand is not fully satisfied [21, 55]

Routes typically start and end at a depot, also located at a vertex of the road graph. A depot is characterised by the vehicle fleet it has at its disposal and the amount of goods it can handle during a given period of time.

A vehicle fleet is located at its home depot, and each vehicle has a load capacity. This capacity may be a weight or volume constraint. A vehicle may be restricted in terms of the routes it takes or the customers it may service. A vehicle also has a utilisation cost associated with it, which may be based on distance, time or route taken.

Knowledge of travel costs and travel times between each pair of customers and between the depot and customers is required in order to evaluate the global cost of routes and to ensure that operational constraints are not violated. In order to achieve this the road graph may be transformed to a complete graph with vertices corresponding to customers and the depot, and with the edges corresponding to shortest paths between all pairs of vertices.

The VRP may be solved for different objectives, such as minimisation of the global transportation cost (depending on global distance or global travel time and on the costs associated with the vehicles used), minimising the number of vehicles required to service all customers, the balancing of routes in terms of travel time and/or vehicle load, and minimising of the penalties associated with partial fulfilment of customer demand. The objective may also be a weighted combination of the above-mentioned objectives.

2.3.2 The Clarke-Wright Algorithm

There are two classes of VRP heuristics, namely classical heuristics and meta-heuristics. Classical heuristics perform a limited exploration of the solution space to produce good solutions within modest computing times. Most classical heuristics are easily extended to account for the various constraints encountered in real-life vehicle routing applications. Meta-heuristics, on the other hand, perform a deep exploration of promising regions of the solution space and produce solutions of a higher quality than classical heuristics. However, the improvement in the quality of solutions comes at the cost of increased computing time.

Dantzig and Ramser [8] introduced the VRP in 1959, proposing a mathematical programming formulation and algorithmic method for its solution. Clarke and Wright [6] offered an improvement to the method of Dantzig and Ramser in 1964. They proposed an effective greedy heuristic solution method which today bears their names.

The Clarke-Wright Algorithm is a classical, constructive heuristic, merging routes according to a savings criterion. The goods to be distributed are considered to be homogeneous and the shortest distances between vertices of the road graph are assumed to be known. The objective is to cover a minimum total distance with all demand being satisfied as loads are allocated to the vehicles.

The Clarke-Wright algorithm applies to problems in which the number of vehicles is a decision variable. Two versions of the algorithm are available, namely a *parallel* and a *sequential* version. The *parallel* version looks for the best feasible merge between all current vehicle routes, while the *sequential* version considers each vehicle route in turn with a view to extend the route by merging it with another route. The *parallel* version was used in this thesis as studies have shown that it dominates the *sequential* version (see, for example, [40]).

A fleet of n vehicles is available for the delivery of goods to m customers. The capacity of the ℓ -th vehicle is assumed to satisfy

$$C_\ell \ll \sum_{k=1}^m q_k, \quad \ell = 1, \dots, n, \quad (2.10)$$

where q_k denotes the volume of or demand for goods to be delivered to customer $k \in \{1, \dots, m\}$ located at vertex P_k from a depot located at the vertex P_0 . It is required to minimise the combined distance covered (or cost incurred) by the vehicle fleet in making deliveries, given the distances $d_{i,j}$ (or cost $c_{i,j}$ incurred) between every pair of positions P_i and P_j .

The Dantzig and Ramser method only considers a single vehicle capacity C_1 . During the first Z stages of the method (r being the current stage) a restriction is enforced that only customers with a combined demand not exceeding a specific fraction $C_1/2^{Z-r}$ of the available vehicle capacity are allowed to be linked on a vehicle route. This typically results in points that are far apart being linked. The emphasis of the method tends more towards filling vehicles to capacity rather than towards minimising the distance travelled by vehicles [6, 8].

During a feasible allocation of deliveries, each customer point is linked to two other points. The depot P_0 may be one or both of these linked points. In order to link at P_i and P_j , four possibilities exist, namely removing the link $P_{i-1}P_i$ or the link P_iP_{i+1} , or joining P_i to P_j where either the link $P_{j-1}P_j$ or P_jP_{j+1} is removed from these routes. These four saving evaluations

are calculated as

$$d_{i,i+1} - d_{0,y+1} + d_{z,z+1} - d_{0,z+1} - d_{i,j}, \quad (2.11)$$

$$d_{i-1,i} - d_{0,y-1} + d_{z,z+1} - d_{0,z+1} - d_{i,j}, \quad (2.12)$$

$$d_{i,i+1} - d_{0,y+1} + d_{z,z-1} - d_{0,z-1} - d_{i,j}, \text{ and} \quad (2.13)$$

$$d_{i-1,i} - d_{0,y-1} + d_{z,z-1} - d_{0,z-1} - d_{i,j}, \quad (2.14)$$

respectively.

The maximum of these evaluations represents the saving $s_{i,j}$ of linking P_i and P_j on a route. Whenever a point is linked to two points on a route with neither of these points being point P_0 , it is no longer considered for linking again [6]. Therefore, the only links that are removed are links from a customer point to P_0 . This results in the saving calculation when linking to points P_i and P_j being reduced to $s_{i,j} = d_{0,i} + d_{0,j} - d_{i,j}$.

If a customer demand q_k is greater than the largest vehicle capacity C_n , then the load can be split into one (or more) full vehicle loads and the remainder of the load. The full loads are serviced exclusively by a vehicle route and q_k subsequently takes the value of the remainder for the rest of the computation.

An initial basic solution is generated by servicing each customer exclusively along a separate route. A route demand variable Q_i represents the total demand of the route containing customer point P_i . A link variable $t_{i,j}$ is defined such that $t_{i,j} = 1$ if P_i and P_j are linked on a vehicle route, or $t_{i,j} = 0$ if they are not linked on a vehicle route. If a customer point P_i is served exclusively by a vehicle route, then $t_{i,0} = 2$, by convention. Due to these conditions, the relationship

$$\sum_{j=0}^{i-1} t_{i,j} + \sum_{j=i+1}^M t_{i,j} = 2 \quad (2.15)$$

exists for all $i = 1, \dots, n$. This relationship ensures that each customer point is visited and departed from only once by some vehicle.

The Clarke-Wright algorithm then orders the savings list in non-decreasing order and proceeds to merge routes should the vehicle constraints allow it (see Algorithm 2.5).

Algorithm 2.5: Clarke-Wright Algorithm

Data: Customer Demand, Distance Matrix, Number of Vehicles

Result: Set of vehicle routes

```

1 for  $i < j$  do
2   for  $j \leq k$  do
3     Calculate  $s_{ij}$ ;
4 Order  $s_{ij} > 0$  in non-decreasing order in  $savings_{list}$ ;
5 while  $savings_{list}$  is not exhausted do
6   if  $t_{i,0} \geq 0$  and  $t_{j,0} \geq 0$  AND
7    $P_i$  and  $P_j$  are not already allocated on the same vehicle route AND
8    $Q_i + Q_j \leq C$ 
9   then
10    Merge route;
11    Set  $Q_i, Q_j \leftarrow Q_i + Q_j$ ;
12    Set  $t_{i,j} \leftarrow 1$  and amend the other values of  $t_{y,z}$  subject to the relationship (2.15)

```

2.4 Geographic Information Systems

A *geographic information system* (GIS) is a computer system that integrates hardware, software and data and is used to capture, store, manage, analyse and display geographically referenced data. Geographically referenced data are also known as geospatial data and describe both the location and characteristics or attributes of spatial features.

2.4.1 Coordinate Systems

A geographic coordinate system is used to reference spatial features in terms of longitude and latitude³ values. When spatial features are displayed on maps they are based on a projected coordinate system in terms of (x, y) -coordinates. Projection is the process of converting a geographic coordinate system to a projected coordinate system in order to display the approximately spherical surface of the earth onto the plane. A reprojection may also be performed to convert a projected coordinate system to another projected coordinate system. The coordinate system in the GIS used for work towards this thesis is the GCS WGS 1984 standard using the D WGS 1984 datum.

2.4.2 Data Models

A data model defines how geospatial data and its spatial features are represented in a GIS. There are two types of GIS data models, the *vector data model* and the *raster data model*.

The vector data model uses (x, y) -coordinates to represent discrete features. The spatial features are represented as points, lines or polygons. These geometric objects and their spatial relationships are organised into digital data files, which can be used to access, interpret and process the data. A point has zero dimension and has only the property of location. A *point feature* is used to represent a point or set of points. A line is one-dimensional and has the properties of location and length. A line consists of two end points and all points on a path between them which mark the shape. The shape of a line may be a smooth curve or a connection of straight line segments. A *line feature* (also sometimes referred to as a *polyline feature*) is used to represent a line or a set of lines. A polygon is two-dimensional and has the properties of location, perimeter and area. A polygon consists of connected, closed non-intersecting line segments. A *polygon feature* is used to represent a single polygon or a set of polygons. In a set context, a polygon may share boundaries with other polygons in the set.

The raster data model uses cells in a grid to represent continuous features. Each cell is defined by its row and column index. In raster data format each cell carries a value, which represents a spatial characteristic at the location denoted by its row and column index. The cell size determines the resolution of the raster. As the cell size decreases, the raster resolution increases, resulting in a more accurate representation. However, the higher resolution also increases data volume and data processing time.

³Longitude and latitude are angular measures. Longitude is the angular measure east or west of the prime meridian and latitude is the angular measure north or south of the equatorial plane. These angular measures of longitude and latitude are expressed in either degrees-minutes-seconds, decimal degrees or radians.

2.4.3 ArcGIS

ARC GIS [16] is a GIS software package produced by Esri [16], a GIS software company. The ARC GIS package used in this research project was ARCVIEW 10.0, which includes ARCCATALOGUE and ARCMAP, the two main applications used. Both ARCCATALOGUE and ARCMAP were used to process data for the analyses described in this thesis. ARCMAP was used to produce the maps to display data geographically in this thesis.

2.4.4 Flowmap

FLOWMAP [18] is a GIS software package developed by Tom De Jong of Utrecht University in the Netherlands in 1990 for the display of flows of goods and people on a map. Since then several tools have been added to FLOWMAP for the management and analysis of spatial relation data, such as distance and flows. These tools are able to perform various tasks on spatial relation data, including computing distances, travel times and transport costs.

2.5 Fitting Data to a Theoretical Statistical Distribution

The software EASYFIT [44] was used in this thesis to fit theoretical statistical distributions to input data. Statistics are computed with respect to the input data, including the sample mean and the sample standard deviation. For each distribution family, the parameters of the distribution are manipulated until the best fit distribution within the given family is found in relation to the input data. The best fit distributions found in all families of distributions may be ranked according to a goodness-of-fit measure. Three goodness-of-fit tests were used in this thesis, namely the *Kolmogorov-Smirnov test*, *Anderson-Darling test* and *Chi-Squared test*.

For a given set of input data, EASYFIT output consisting of three components is given: (a) a graph for each best-fitting distribution to visually compare the distribution with the input data, (b) a summary of the parameters for each best-fitting distribution and, (c) a list with the best-fitting distributions ranked for the three goodness-of-fit tests with a link to the test statistics for the goodness-of-fit tests comparing how well the input data fitted the best-fitting distribution at different levels of significance.

2.5.1 The Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test is used to decide whether a sample is from a hypothesized continuous distribution and is based on the empirical cumulative distribution function of the sample. Assuming a random sample x_1, \dots, x_n from some distribution with a cumulative distribution function $F(x)$, the empirical cumulative distribution function is

$$F_n(x) = \frac{1}{n} \cdot [\text{Number of observations} \leq x].$$

The Kolmogorov-Smirnov statistic is defined as

$$D = \max_{1 \leq i \leq n} \left\{ F(x_i) - \frac{i-1}{n}, \frac{i}{n} - F(x_i) \right\}$$

and is based on the largest vertical difference between the theoretical and the empirical cumulative distribution function.

The null and alternative hypotheses are

H₀: the data follow the specified distribution, and

H_A: the data do not follow the specified distribution,

respectively.

At a chosen significance level α , the null hypothesis is rejected if D is greater than the critical value obtained from the test table [20].

A so-called *P-value* is calculated, based on D , and denotes the threshold of α . **H₀** is not rejected for all values of α less than the *P-value* [44].

2.5.2 The Anderson-Darling Test

The Anderson-Darling test is used to compare the fit of an observed cumulative distribution function to an expected cumulative distribution function. This test gives more weight to the tails of the distribution than does the Kolmogorov-Smirnov test.

The Anderson-Darling statistic for a sample x_1, \dots, x_n is defined as

$$A^2 = -n - \frac{1}{n} \sum_{i=1}^n (2i - 1) \cdot [\ln F(x_i) + \ln(1 - F(x_{n-i+1}))].$$

The null and alternative hypotheses are

H₀: the data follow the specified distribution, and

H_A: the data do not follow the specified distribution,

respectively.

At a chosen significance level α , the null hypothesis is rejected if A^2 is greater than the critical value obtained from the test table [20].

In general, critical values of the Anderson-Darling test statistic depend on the specific distribution being tested. However, in EASYFIT the Anderson-Darling test is implemented using the same critical values, calculated by means of an approximation formula depending on sample size only, for all distributions. This implementation of the Anderson-Darling test results in a less likely rejection of a good fit and can be used to compare the goodness-of-fit of several fitted distributions [44].

2.5.3 The Chi-Squared Test

The Chi-Squared test is used to determine whether a sample comes from a population with a specific distribution. This test is applied to binned data, and so the value of the test statistic depends on how the data are binned. This test is available for continuous sample data only.

There is no optimal choice for the number of bins, k . However, there are formulas which may be used to calculate a desirable number of bins based on the sample size n . EASYFIT uses the empirical formula

$$k = 1 + \log_2 n.$$

The data may be grouped into equal probability or equal width intervals, with the equal probability width bin approach being the more widely accepted method since it handles peaked data

better [44]. Each bin should contain at least five data points, resulting in certain adjacent bins being joined together to satisfy this condition.

The Chi-Squared statistic is defined as

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i},$$

where O_i is the observed frequency of data in bin i , and E_i is the expected frequency of data in bin i , calculated as $E_i = F(y_2) - F(y_1)$, where F is the cumulative distribution function of the probability distribution being tested, and y_1, y_2 are the limits for bin i .

The null and alternative hypotheses are

H₀: the data follow the specified distribution, and

H_A: the data do not follow the specified distribution.

respectively.

At a chosen significance level α , the null hypothesis is rejected if the test statistic is greater than the critical value $\chi^2_{1-\alpha, k-1}$, denoting the Chi-Squared inverse cumulative distribution function with $k - 1$ degrees of freedom and significance level of α [37].

The number of degrees of freedom is $k - c - 1$, where c is the number of estimated parameters. EASYFIT, however, uses $k - 1$ degrees of freedom as it is less likely to reject the fit in error.

The P -value is calculated based on the χ^2 -value and denotes the threshold of α in the sense that **H₀** is not rejected for all values of α less than the P -value [44].

2.6 Chapter Summary

In §2.1, three facility location problems were reviewed, namely the K -Centre Problem, the K -Median Problem and both the capacitated and uncapacitated models of the Fixed Charge Location Problem.

Solution methods for linear and integer programming models were also described in §2.2, including the simplex and dual simplex algorithms, the branch-and-bound method and the classical tabu search method. The solution approach adopted by LINGO 11.0 were described and the specialised CCP-ATS algorithm for solving facility location problems approximately was explained.

The basic vehicle routing problem was reviewed and the well-known Clarke-Wright algorithm was described in §2.3. This was followed in §2.4 by an explanation of the notion of GIS, considering the coordinate systems and data models most often used. The two GIS software packages, namely ARCGIS and FLOWMAP, used in this thesis were also touched upon. The process of fitting data to a theoretical statistical distribution was considered in §2.5 and the software EASYFIT used for fitting the data to theoretical distributions in this thesis was explained, including a review of the three goodness-of-fit tests used by EASYFIT.

CHAPTER 3

Facility Location Problem

Contents

3.1	Problem Description and Methodology	29
3.1.1	<i>Background Information</i>	30
3.1.2	<i>Food Distribution Scenarios</i>	32
3.1.3	<i>Geographical Information System Data</i>	34
3.1.4	<i>Infrastructure Location</i>	34
3.1.5	<i>Demand Data</i>	35
3.1.6	<i>Candidate Sites</i>	35
3.1.7	<i>Distance and Cost</i>	39
3.2	Model Formulation	43
3.3	Application to Test Problems	44
3.3.1	<i>Generation of Test Problems</i>	44
3.3.2	<i>Numerical Results</i>	45
3.4	Application on FBCT Data and Results	50
3.4.1	<i>Application on FBCT Data</i>	50
3.4.2	<i>Results</i>	52
3.5	Chapter Summary	58

The *Facility Location Problem* (FLP) of FBCT is concerned with determining the location of foodbank depots. These depots act as drop-off points to which FBCT deliver food and from which the agencies collect food. The objective of the FLP is to minimise the actual distance travelled during the process of distribution of the food or to minimise the distribution cost. The total distribution cost (or distance travelled) has two components: the cost to FBCT and the cost to agencies. In this chapter the FLP is considered in terms of distance, while the FLP in terms of cost will be addressed in the following chapter.

3.1 Problem Description and Methodology

For an FLP the type of facilities that are being located and the factors that influence the location have to be specified. In the FBCT FLP the facilities being located are local distribution depots and the main factors influencing the location are the demand for food by agencies assigned to these depots and the distance travelled or cost incurred for the distribution of food to satisfy

this demand. The demand of an agency is the food that it requires or has been assigned to the given agency. The distance travelled or cost incurred has two components, the component FBCT are responsible for and the component the agencies are responsible for.

In order to solve the FBCT FLP, the following tools and data are required: warehouse, agency and candidate depot locations, a system to map these locations, agency demands and a method to calculate the travel costs between any two locations. As described in the previous chapter (§2.1), a problem such as this may be modelled by means of a graph in which the vertices represent the warehouse, agency and candidate depot locations and in which the edges represent the travel routes between these infrastructure components, with edge weights representing the distances or costs of travelling along the edges. The warehouse may be seen as the supply point, the agencies as the set of demand points and the candidate depot locations as the set of candidate facility sites.

There are a number of questions that are regarded as important to consider before and during the solution of the FLP. These questions are addressed in both this chapter and the next, and include the following:

1. Which candidate sites would be suitable as depots? This question is addressed later in this section.
2. What is the preferred number of depots? It was suggested by FBCT that an average of between 5 and 10 agencies per depot would be preferable [10].
3. What percentage of the total cost of distribution (or distance travelled) would FBCT be prepared to carry? The breakdown of the cost of distribution between FBCT and the agencies is considered highly valuable information to FBCT.

These are important considerations for FBSA and FBCT and an objective of the work in this thesis is assisting FBCT by using model results to facilitate the organisation in answering these questions. Questions 2 and 3 are also listed as Objectives $V(a)$ and $V(c)$, in §1.3.2, respectively.

3.1.1 Background Information

The process of distribution of food to agencies was targeted as the main focus of this thesis. One of the difficulties in isolating this distribution process from the other operations of FBCT is that the same fleet of vehicles is used for both the sourcing and distribution of food. A system which takes into account both pickups and deliveries together initially seemed to be the best approach towards solving the problem. However, there are factors which complicate such an approach.

The composition of the suppliers is expected to change in the future. Currently most food is supplied from retailers and *distribution centres* (DCs) and consist of food items that are taken off the shelves due to damaged packaging or the fact that they are close to their expiry dates. FBSA is, however, in the process of developing strategic partnerships with food manufacturers and role players in the agricultural industry [25, 38]. In the future food will be procured, possibly at a reduced cost, from these two industries. Two benefits of such partnerships are a more steady influx of food and the fact that FBSA would have a say in the type of food that is purchased. Food currently supplied by retailers and DCs is not always in line with what constitutes a balanced, nutritious meal. However, food types procured from manufacturers and the agricultural industry would typically be strategically planned, allowing FBSA to deliver

balanced, nutritious meals to the agencies. Food procured as a result of these partnerships would be supplemented by the food that is currently received from retailers and DCs. It is thought that the financial costs involved in sourcing food from manufacturers and agricultural industry will be lower per food volume, although the processing, sorting and cleaning operations involved may require more time and labour for FBSA. Another benefit may be that the shelf life of food in the foodbank warehouse is expected to increase, as it is assumed that the food products from manufacturers and the agricultural industry would be fresher when received than those currently received from retailers and DCs.

Furthermore, a large portion of the food currently collected from suppliers is collected by FBCT after hours and may therefore be considered as separate pick-up trips that are independent of trips for other operations or purposes. Deliveries have to be made to agencies during working hours; therefore, these pick-ups and delivery trips fall into different time frames. The larger and more consistent suppliers fall in the after hours pick-up category. The other suppliers are not as consistent and the times, and hence days, for pick-ups are more uncertain. A supplier typically calls (sometimes on the day in question and other times a few days in advance) to let FBCT know that there is a food parcel waiting or will be available for collection at a certain time. This has been noted as a problem with the coordination of daily routes for vehicles, as a route for the day will typically already have been planned out when such a call is received by FBCT and then the driver is contacted and instructed to divert from the planned route to collect from a supplier. If the pick-up is known at the beginning of the day, then the route can be adjusted in advance; however, it does restrict the planning of delivery routes to daily planning and does not allow much scope for weekly route planning.

Two members of FBCT staff [42, 46] involved in the planning of routes and vehicle management suggested that separating the pick-up and delivery roles would be beneficial to them in the route planning roles that they fulfil. They would then be able to put a more structured set of weekly routes in place for deliveries and would only need to focus on routing the pick-ups as required. (This had been suggested to higher management. However, without an estimated level of improvement to the system it has not been viewed as a viable option as yet. It is nevertheless hoped that the results of quantitative models such as the one in this thesis may convince FBSA and FBCT management to accept the recommendation of their vehicle route planning staff.)

All¹ sourced food must first be taken to the warehouse to be cleaned and sorted. The idea is that depots would act only as drop-off points during the distribution process; no food would be stored there. Therefore, the locations of depots are more directly influenced by the proximity of agencies than they are to the proximity of suppliers.

For the above reasons it was decided to consider the process of food distribution as a separate aspect of the FBCT's operations. This resulted in the assumption that vehicles for the distribution and collection of food would be viewed as forming separate vehicle fleets.

¹The exception to this rule is the Lunch Buddy program [23, 24], in which donor schools partner with under-resourced schools. Pupils at the more resourced donor school bring an extra sandwich to school and donate it to the Lunch Buddy program. FBCT collects these sandwiches and delivers them directly to the under-resourced schools which distribute these sandwiches to their pupils.

3.1.2 Food Distribution Scenarios

The distribution of food to agencies may conform to one of three possible scenarios:

1. The agency collects directly from the FBCT warehouse.
2. FBCT delivers directly to the agency.
3. FBCT delivers the food to a depot and agencies assigned to the depot then collect the food from the depot.

If all agencies collect from the FBCT warehouse, as depicted in Figure 3.1, the financial burden of the distribution falls entirely to the agencies. It also requires that each agency should individually collect food and the travelling distances involved may cause a financial concern for some of these social service agencies. This scenario is also not ideal for FBCT either as the agencies will arrive at the warehouse unannounced, causing congestion at its loading bays.

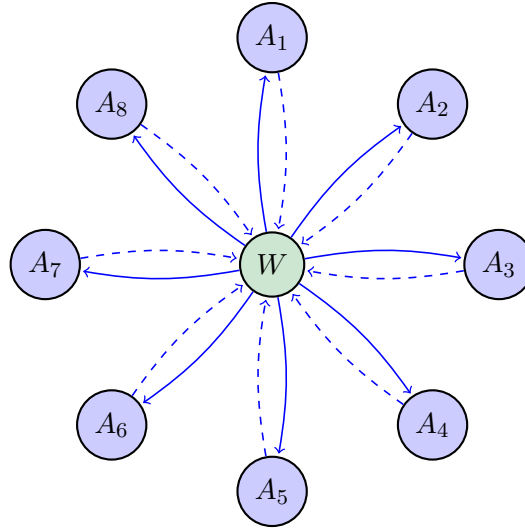


Figure 3.1: The situation where agencies (A_1, \dots, A_8) collect food directly from the FBCT warehouse (W). The portions of routes during which agency pick-up vehicles carry food items (are empty, resp.) are denoted by solid (broken, resp.) arcs.

If, on the other hand, FBCT delivers to all agencies individually, as depicted in Figure 3.2(a), the financial burden of the distribution falls entirely to FBCT. This situation is not ideal for FBCT as delivering to each agency will also put strain on their very limited resources, such as time, vehicles and staff. The application of vehicle routing is expected to assist in this regard by reducing the distances travelled or costs incurred in delivering food to all the agencies, as depicted in Figure 3.2(b).

If depots are used in the distribution of food, as depicted in Figure 3.3(a), the financial burden of the distribution is more evenly spread between FBCT and the agencies. This requires finding locations for the depots and should allow time for the process of establishing the depots. Decisions on staffing and the cost involved in establishing the depots also need to be taken into account. The application of vehicle routing is again expected to assist in reducing the distances travelled or costs incurred in delivering food to the depots, as depicted in Figure 3.3(b).

During meetings with FBCT and FBSA staff, it was found to be very important for FBCT not to have to deliver to each agency separately and thus incur the entire cost of food distribution

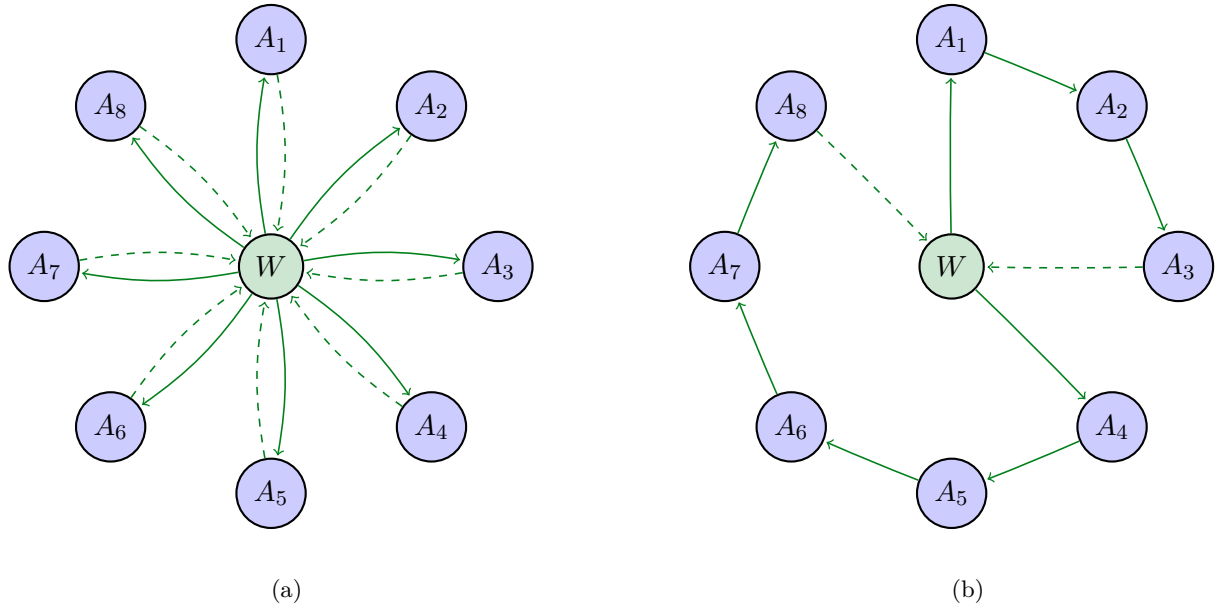


Figure 3.2: (a) The situation where the FBCT warehouse (W) delivers food directly to agencies (A_1, \dots, A_8). The portions of routes during which FBCT delivery vehicles carry food items (are empty, resp.) are denoted by solid (broken, resp.) arcs. (b) The same situation as in (a), but with vehicle routing methods applied to the FBCT vehicle routes.

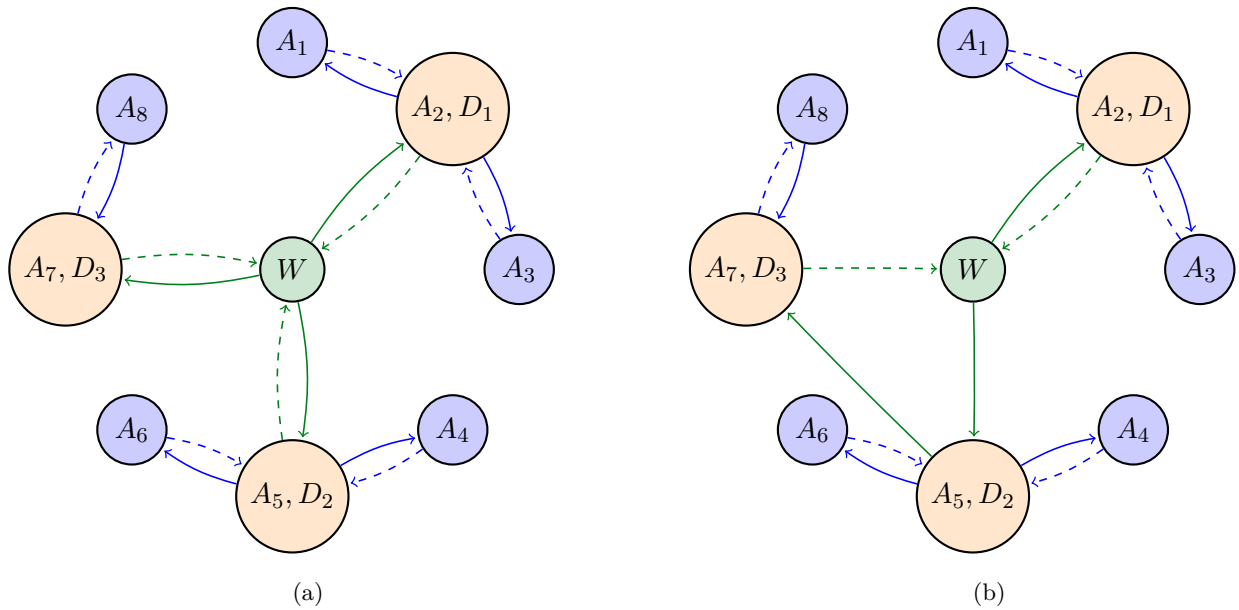


Figure 3.3: (a) The situation where the FBCT warehouse (W) delivers food to depots (D_1, D_2, D_3) and the agencies (A_1, \dots, A_8) collect food from depots. The portions of routes during which agency pick-up or FBCT delivery vehicles carry food items (are empty, resp.) are denoted by solid (broken, resp.) arcs. (b) The same situation as in (a), but with vehicle routing methods applied to the FBCT vehicle routes.

themselves. It is also unrealistic to expect all agencies to collect food from the warehouse as this would put both the agencies and the warehouse facilities under considerable pressure (both in terms of cost and congestion). Therefore, the scenario of establishing depots was deemed the best scenario to adopt for the distribution of food.

3.1.3 Geographical Information System Data

Geographical Information System (GIS) shapefiles compatible with the ArcGIS [16] and Flowmap [18] software suites were obtained from Mans [43] and Stephenson [54]. These files include a shapefile of the City of Cape Town broken down into hexagons (the file called `cpthex_pop2009.shp` on the compact disc accompanying this thesis), a shapefile of the road network of the City of Cape Town (the file called `Cpt.Roads.shp` on the compact disc accompanying this thesis), a shapefile of the national land use for the City of Cape Town (the file called `nlc_.shp` on the compact disc accompanying this thesis) and shapefiles of the 1:50 000 maps of the Western Cape (e.g. the file called `3418BA_2000_ED7_GEO.TIF` on the compact disc accompanying this thesis). The road network shapefile is in vector data model format and consists of a series of line segments joined together. The length of a line segment represents the distance along the corresponding road segment. The hexagon shapefile is also in vector data model format in which the City of Cape Town area is broken down into 6 543 regular hexagons, each with an area of 40 hectares (0.4 square kilometres) and with embedded population data. The national land use shapefile contains information on urban areas broken down into polygons representing residential, commercial, industrial and smallholding area usages. This file is also in vector data model format. The 1:50 000 maps are in raster data model format and were used to verify that features mapped in ARCMAP were accurate.

The midpoint of each hexagon was calculated and used as the location point for any FBCT infrastructure component that fell within the hexagon. The furthestmost point from the midpoint of a hexagon to any point within the hexagon is the same as the length of a side of the hexagon. This results in no point in the hexagon being further than approximately 310 metres from its midpoint (the road distance may, however, be greater). All distances between hexagons were measured from midpoint to midpoint, using the road segment closest to the midpoint.

3.1.4 Infrastructure Location

Agency-related data were received from FBCT in the form of two excel spreadsheets (the files called `DATABASE_CT.xls` and `GPS_Locations.xlsx` on the compact disc accompanying this thesis). The former file contains the physical addresses of agencies, while the latter contains the *Global Positioning System* (GPS) coordinates of agencies used by FBCT in their C-Track vehicle tracking system [12]. Not all agency addresses were entered on the C-Track system. Using the addresses in the file containing the physical addresses of agencies, the coordinates of these agencies were found using Google Maps [30]. Initially there were just under 300 agencies altogether in both lists; this is less than the 360 agencies that FBCT started servicing at the beginning of their operations [3]. The lists were correlated and discrepancies were corrected². Once the lists were combined there were still a few agency details that were incomplete. Communication with Davison [10] led to the updating of these agency details or removals from the agency list if agencies were no longer being serviced by FBCT³. Once the list was up to date there were approximately 200 agencies which were being serviced; this corresponds with an excel spreadsheet received mid 2011 which had been used by Watson in his masters thesis [57], focussing on the design of a food allocation system for FBSA. The agencies, agency hexagons, warehouse

²Discrepancies included spelling mistakes in agency names and addresses, not enough address detail and wrong addresses given.

³Agencies are regulated by FBCT and if an agency no longer satisfies certain requirements put in place by FBCT, service to it may be discontinued. This is a continual process. At the beginning of 2010 some agencies that had been carried over from the amalgamation were still being evaluated.

hexagon and the warehouse were mapped (see Figures 3.4–3.6) using the GIS software suite ARCMAP produced by Esri [16]. The locations of the agencies and warehouse may be seen in Figure 3.4.

It was decided to reduce the size of the problem by regarding all agencies within any one hexagon as a single agency. The maximum number of agencies in any one hexagon was four and this only occurred for one hexagon. The demands of the agencies in the same hexagon were aggregated, yielding a demand for each agency hexagon. This reduced the number of agencies to 157 agency hexagons (see Figures 3.5 and 3.6). The largest delivery vehicle owned by FBCT, has a carrying capacity of 5 000 kilograms. Therefore, the only agency hexagon that had a demand of over 5 000 kilograms was regarded as two agency hexagons of equal demand, located at the same place. This resulted in 158 agency hexagons being used as demand points in the FLP.

The coordinates of the FBCT warehouse were also received from FBCT and verified against the coordinates of the address on Google Maps.

3.1.5 Demand Data

Initially it was hoped that detailed supply data on the allocation of food to agencies would be available over a relatively long period of time (preferably between six months to a year). This would have been used to construct expected/predicted demand for each of the agencies. Unfortunately, very little data were obtained and those that were obtained were in a non-user friendly format⁴.

Accurate demand data for the agencies were obtained from an excel spreadsheet (the file called FAST information.xlsx on the compact disc accompanying this thesis) provided by FBCT. For each agency a *total beneficiaries* value was provided, representing the number of people the agency supports as well as the *number of meals per week* that the agency supplies to these beneficiaries. FBCT used these figures to determine the *adjusted number of meals per week* each agency should receive. A meal was considered to be 0.3 kilograms of food. This was used to generate an *aim to allocate kg per week* value for each agency. The total of *aim to allocate kg per week* is 83 592.3 kilograms. Although these data did not give current allocations, they represent projected allocations that FBCT would like to fulfil in the medium/long term. Since the improvement of distribution logistics and focussing on depot location was seen as a medium term project, it was decided to use these projected allocations as the demand for agencies. These data were therefore used under the assumptions that ultimately enough food would be sourced to supply the agencies with this *aim to allocate kg per week* amount of food and that all food allocated to an agency would be available for its required day of delivery. Also, new agencies would not be added to the list of agencies until each current agency was receiving the amount of food equivalent to its *aim to allocate kg per week* value.

3.1.6 Candidate Sites

During early conversations with various members of the FBCT staff [1, 10, 32] every suitable hexagon⁵ would be considered as a potential depot location. However, the fixed cost for establishing a depot varies from hexagon to hexagon, depending on the land use of the hexagon, its

⁴Incomplete entries and unsorted data with PivotTable reports without underlying data.

⁵Hexagons that fell within residential and commercial land use areas, obtained from the national land cover GIS shapefile (see Figure 3.7).

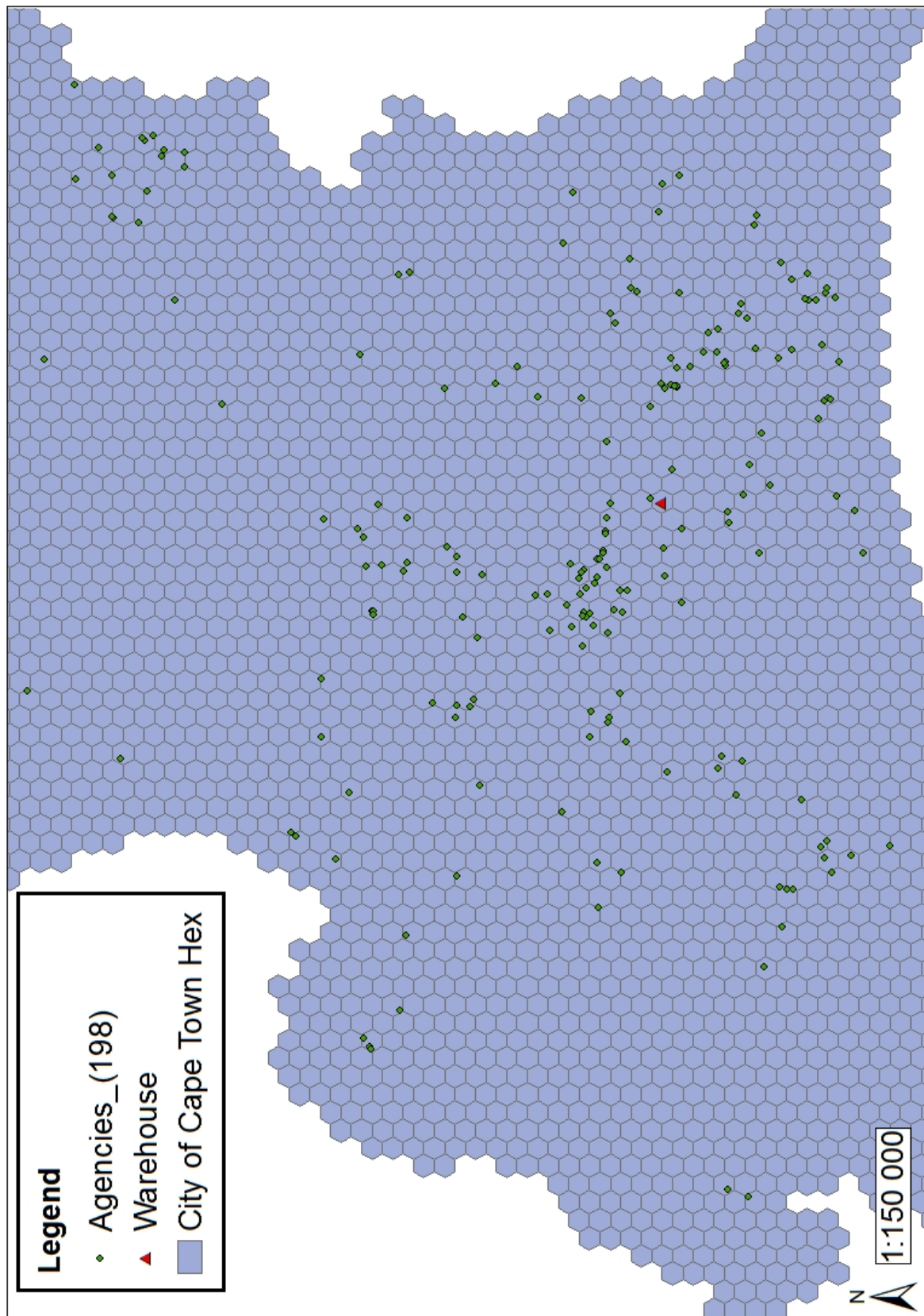


Figure 3.4: The FoodBank Cape Town warehouse and the 198 agencies they service, mapped in ArcMAP [16].

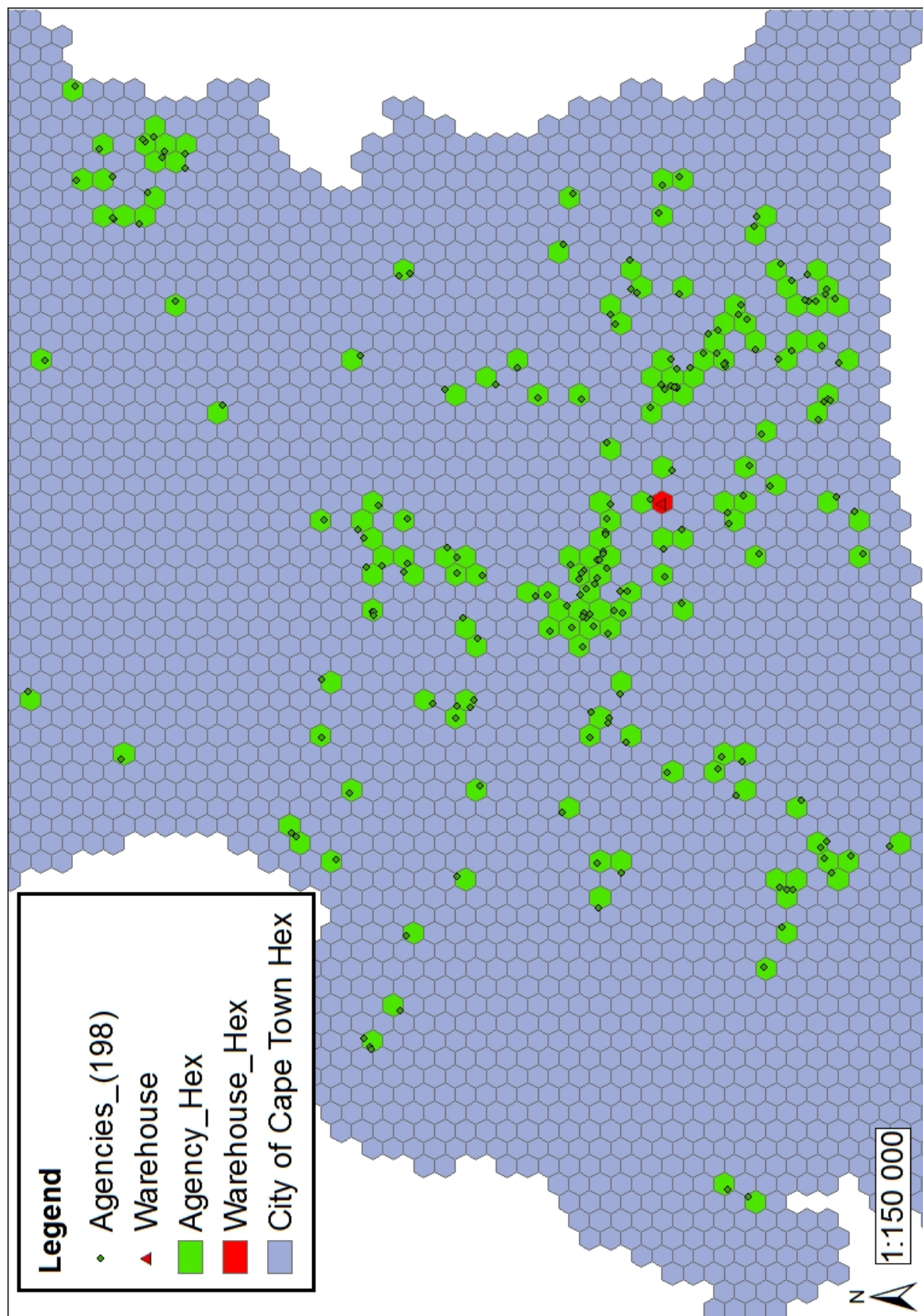


Figure 3.5: The FoodBank Cape Town warehouse and agencies, and the hexagons they fell within, mapped in ARCMAP [16].

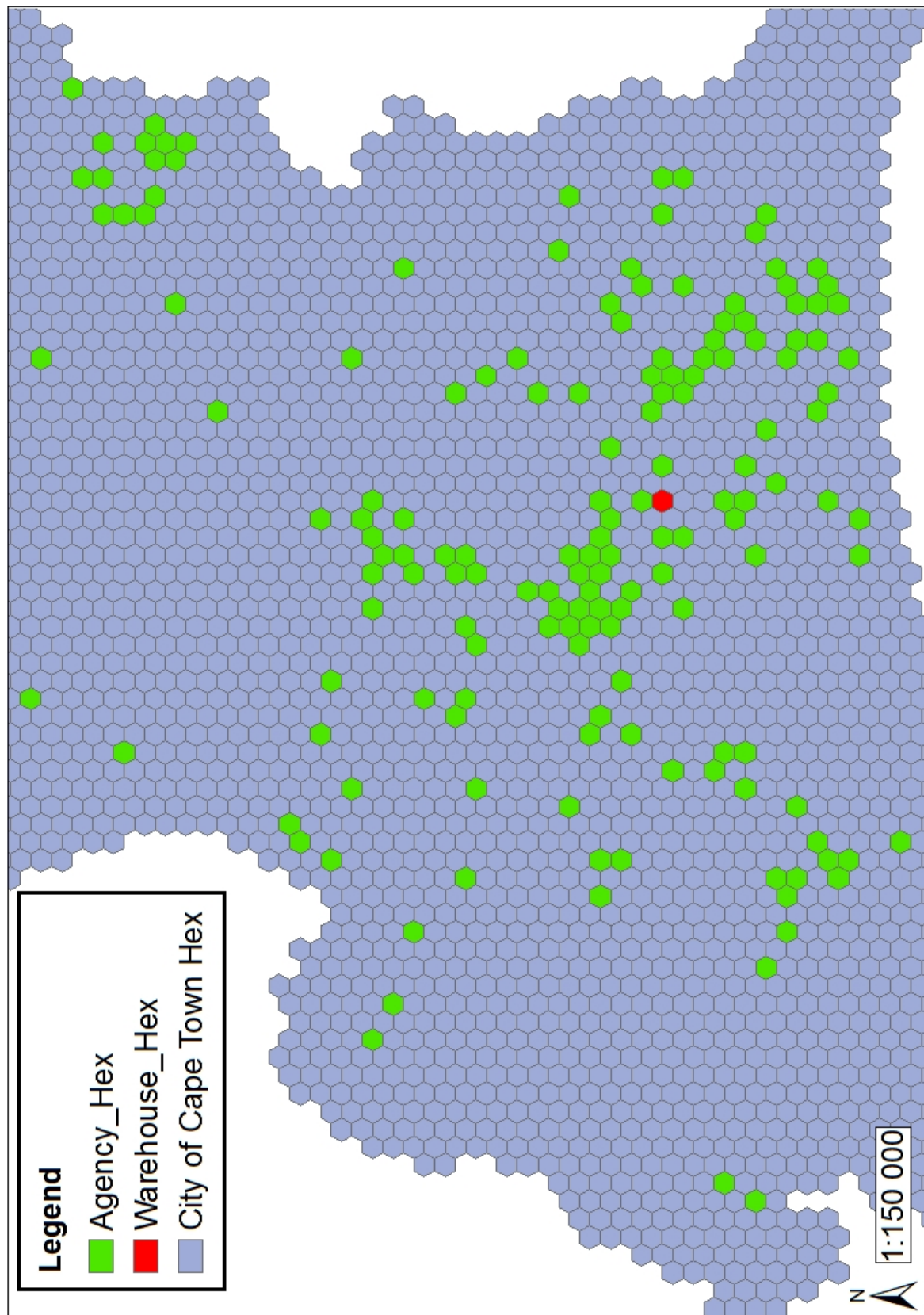


Figure 3.6: The FoodBank Cape Town warehouse hexagon and the 157 agency hexagons, mapped in ARCMAP [16].

average property value and the size of available property within a hexagon. The land use areas of the City of Cape Town may be seen in Figure 3.7.

This reduced the potential candidate sites to 1 763 hexagons (see Figure 3.8). The number of candidate sites was further reduced to 1 169 hexagons by removing those hexagons that were not within the central area which contains agencies (see Figure 3.9).

However, during the beginning of 2011 this view of FBCT changed with the result that only agencies would be considered as candidate locations for depots. In this way the cost for establishing a depot would be negated as the depot would be run by the agency at no financial cost to FBCT⁶. The set of candidate depot sites was therefore the same as the agency hexagons set (see Figure 3.6).

3.1.7 Distance and Cost

A simple basis for calculating transportation costs is to multiply the distance travelled by a cost per distance unit value. It is therefore important to be able to determine the cost per distance unit value and calculate the distance travelled accurately. Two options are available for the latter: using actual distances along the *road network* or using the *Euclidean distance multiplied by an air-to-road factor*⁷.

As mentioned, a *road network* in a GIS shape file format consists of a series of line segments joined together, implemented in GIS software. The length of a line segment represents the distance along the corresponding road segment. Due to it being implemented in a GIS format, speed limit data or other data deemed as important may also be attached to each segment. The distance between any two points in the network can be easily calculated, but the process of creating such a network in the first place is time-consuming and requires extensive GIS expertise. The calculation of pairwise distances between a large set of points may also require long computation times. Fortunately such a road network for the City of Cape Town had already been created and was readily available. Permission to use this road network in the current project was granted by the Head of Information and Research the City of Cape Town [56]. The Flowmap [18] GIS software suite was used to calculate road distances with the assistance of Mans [43].

The *Euclidean distance multiplied by an air-to-road factor* method requires the coordinates of two points so as to calculate the Euclidean distance between them. The Euclidean distance is then multiplied by an approximate air-to-road factor in order to obtain the approximate road distance between the two points. The air-to-road factor value of 1.3 is acceptable for the City of Cape Town region [3]. Using this method the calculation of distances between a large set of points would be a simpler and less time consuming process.

It was, however, decided rather to adopt the *road network* approach as it produced a more accurate calculation of the distance between locations and would provide realistic representation of the routes that FBCT may use between locations.

⁶The agency would benefit from “hosting” the depot by not having to collect food as the food would be delivered to their premises. The agency concerned would also benefit from a stronger relationship with FBCT.

⁷This factor is typical of the terrain and is an estimate of the ratio of road distances between various points in the area to Euclidean distances between the same points.

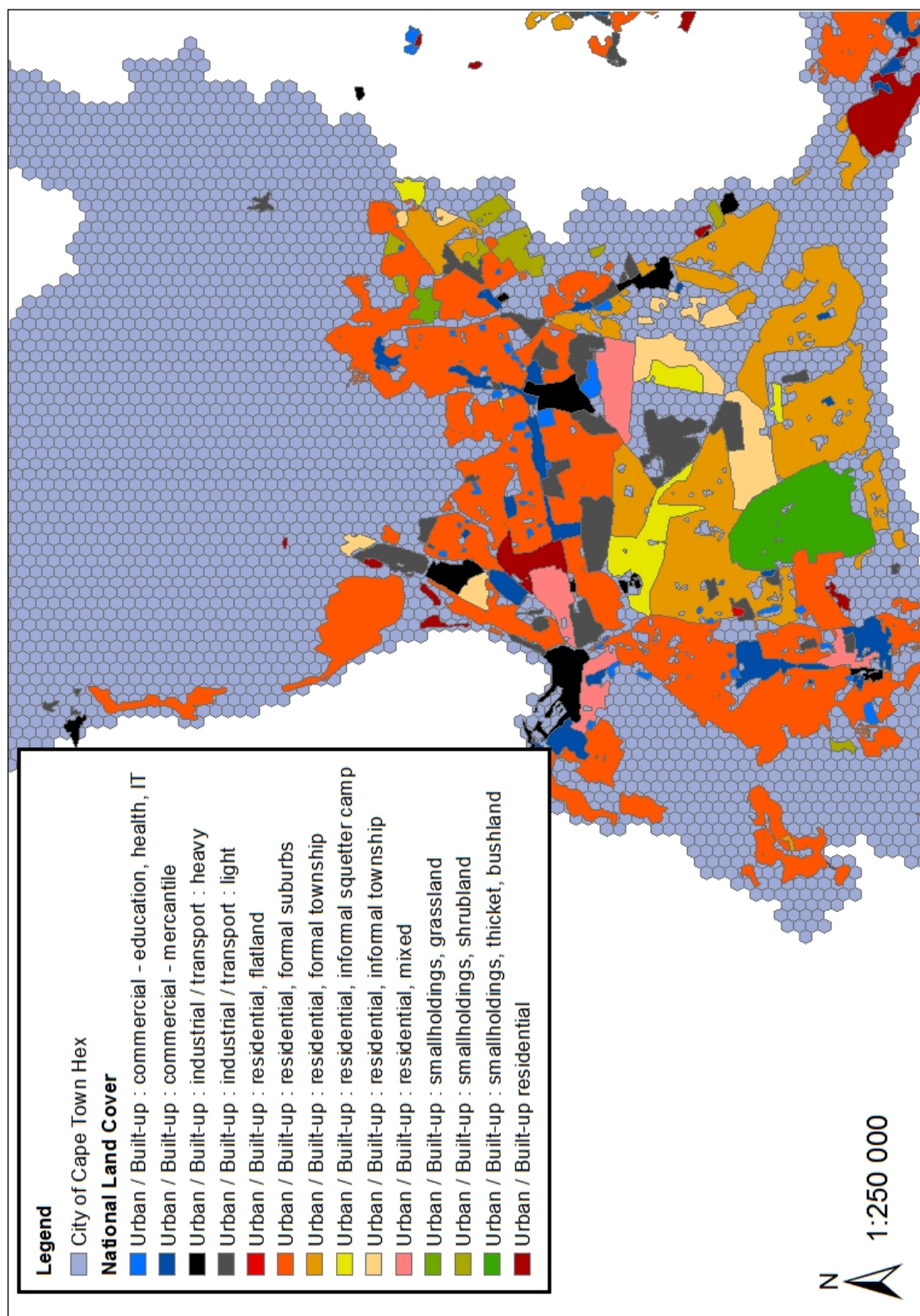


Figure 3.7: National land use for the City of Cape Town, mapped in ARCMAP [16].

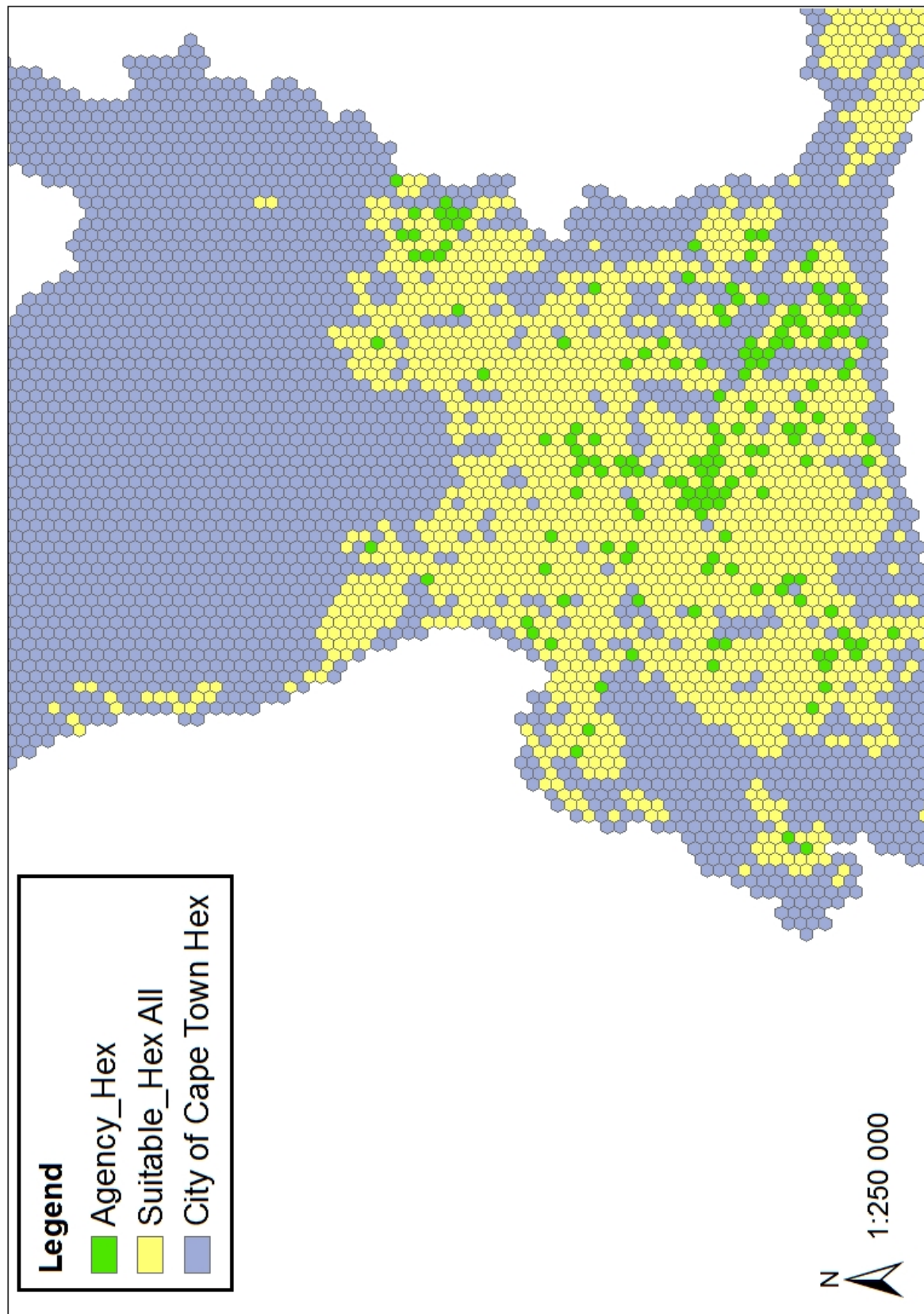


Figure 3.8: The 1 763 hexagons that were originally suitable as candidate location hexagons, mapped in ArcMAP [16].

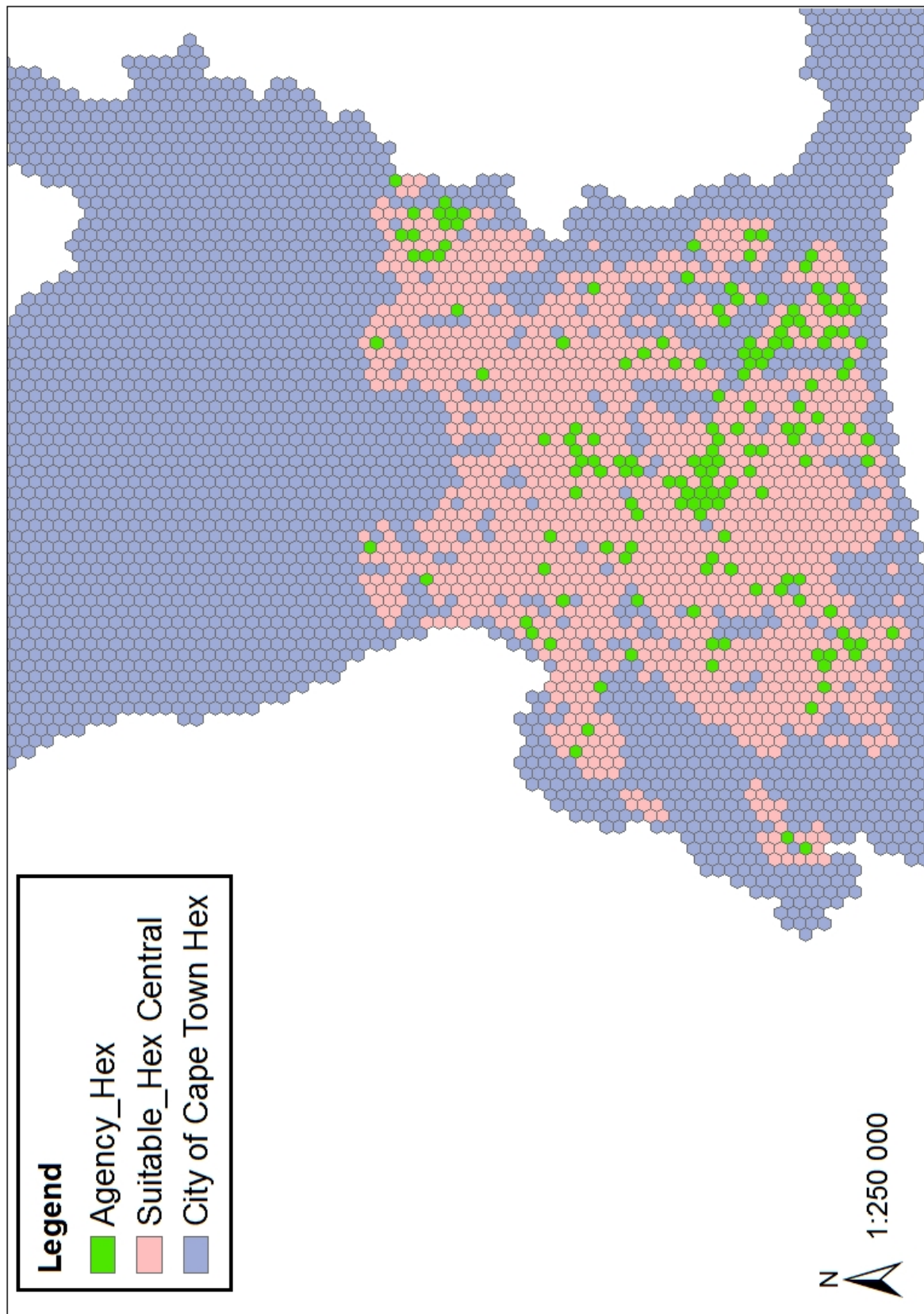


Figure 3.9: The 1 169 hexagons that were to be used as candidate location hexagons, mapped in ARCMAP [16].

3.2 Model Formulation

An FLP model is required to determine the location of depots to service the 158 agency hexagons. A total demand of 83 952.3 kilograms would need to be distributed to the depots, with each depot having a capacity constraint of 5 000 kilograms⁸.

Initially an FLP model for minimising the distance (proportional to cost, when all vehicles costs are the same) that agencies would travel to their assigned depots was considered. This model would have resembled the *K-median problem* model. However, the distance that the FBCT delivery vehicles travel in order to deliver the food to the depots needed to be incorporated.

The FBCT FLP model based on *agency-depot* (AD) distance and *warehouse-depot* (WD) distance is formulated as a mixed-integer linear programming problem, with the objective of minimising the total distance

$$g = \sum_{j=1}^m d_{0j}x_j + \sum_{i=1}^n \sum_{j=1}^m d_{ij}y_{ij} \quad (3.1)$$

between the warehouse and each depot and each agency and its assigned depot, where d_{0j} is the distance from the warehouse to depot $j \in J$ and d_{ij} is the distance from agency $i \in I$ to candidate depot location $j \in J$, and where

$$y_{ij} = \begin{cases} 1 & \text{if the agency } i \text{ is assigned to depot } j, \\ 0 & \text{otherwise} \end{cases}$$

is a binary decision variable. Upon defining the additional binary decision variable

$$x_j = \begin{cases} 1 & \text{if depot is located at vertex } j, \\ 0 & \text{otherwise,} \end{cases}$$

the minimisation of g in (3.1) should be carried out subject to the constraints

$$\sum_{j \in J} x_j = K, \quad j = 1, \dots, m \quad (3.2)$$

$$\sum_{j \in J} y_{ij} = 1, \quad i = 1, \dots, n \quad (3.3)$$

$$y_{ij} \leq x_j, \quad i = 1, \dots, n \\ j = 1, \dots, m \quad (3.4)$$

$$\sum_{i \in I} h_i y_{ij} \leq C_j x_j, \quad j = 1, \dots, m \quad (3.5)$$

$$x_j, y_{ij} \in \{1, 0\}, \quad i = 1, \dots, n \\ j = 1, \dots, m \quad (3.6)$$

where K denotes the number of depots, h_i denotes the demand of agency i and C_j denotes the capacity of depot j . Here the constraint set (3.2) ensures that exactly K facilities are located, while the constraint sets (3.3) and (3.6) ensure that demand of an agency is assigned to one depot. Constraint set (3.4) ensures that demand is only assigned to a depot which has actually been located and constraint set (3.5) ensures that the demand assigned to a depot does not exceed the capacity of the depot.

⁸Five thousand kilograms is the capacity of the largest delivery vehicle used by FBCT.

This model resembles the *fixed charge location problem* model (see §2.1). The distance that FBCT travels to each depot location may be viewed as the fixed charge of having a depot at that location, as each week this distance will be travelled by the FBCT delivery vehicles. Distance, and not demand weighted distance, is the value being minimised and each depot is capacitated. The reason for choosing to minimise distance, and not demand weighted distance, is that regardless of the quantity of the demand, an agency will still have to travel the same distance to collect the food allocated to it. It is assumed that each agency will have a vehicle large enough to collect all food destined for that agency in one trip. This model does not take into account the distance that may be saved as a result of vehicle routing being applied to the FBCT vehicles or the extra distance that may have to be travelled if a smaller FBCT vehicle is used and has to perform multiple trips to a depot. This shortcoming is, however, addressed in the next chapter when the cost of distribution is dealt with.

If the assumption that agency and FBCT costs are equal over the same distance were to be made, a simple cost of the distribution may be obtained by simply multiplying the distance calculated by a cost per distance factor. However, the cost of distribution is dealt with in the next chapter, as was mentioned above.

3.3 Application to Test Problems

Before incorporating all the FBCT data into the model (3.1)–(3.6) it was decided to first test the model with respect to smaller data sets. Data were generated to test the ability of LINGO 11.0 [41] to solve the model and to test the effectiveness of using a *Capacitated Clustering Problem Adaptive Tabu Search* (CCP-ATS) to solve the model. The data were generated randomly to imitate the actual FBCT data, but for smaller problem dimensions.

3.3.1 Generation of Test Problems

Various instances of three sizes of problems were generated in order to test the effectiveness of the two solution methodologies for the FPL model mentioned above. Problems having 25, 50 and 100 agencies were considered. For each problem size 30 instances were generated. The depot capacity was set at 5 000 demand units (to correspond to the capacity of depots for the FBCT problem) in each case. The agencies were located in the euclidean plane, with their x and y coordinates randomly generated from a uniform distribution between 0 and 50 distance units⁹. The Euclidean distance between agencies were calculated and used as the travel distances between agencies. The agency demands were generated to represent a similar profile to that of the FBCT data. Seven ranges and an approximate percentage of agencies to fall in each range were determined. For each range the demand values were randomly generated from a uniform distribution between the minimum and maximum values of the range, as per Table 3.1.

As a problem instance becomes less constrained LINGO 11.0 tends to solve the problem quicker. Therefore, to test the capability of LINGO 11.0 with respect to solving these problem instances quickly, the value of K (number of depots) was set as low as feasibly possible in order to render the problem highly constrained. A *tightness value*, τ , was used as a measure of how highly constrained a problem instance is. For the test instances the tightness value of τ was kept between $[0.82, 0.96]$, corresponding to the effective τ tightness factor for the CCP-ATS

⁹ For the FBCT data the average maximum distance from an agency to another agency was approximately 35 kilometres, so this distance scale is similar to the distance scale of the FBCT data.

Demand <i>min</i>	Demand <i>max</i>	FBCT Agencies (158)	Approximate percentage	Agencies Size 25	Agencies Size 50	Agencies Size 100
0	100	35	20	5	10	20
100	200	32	20	5	10	20
200	300	15	10	3	5	10
300	500	32	20	5	10	20
500	1 000	23	16	4	8	16
1 000	2 500	14	10	2	5	10
2 500	4 750	7	4	1	2	4

Table 3.1: The profile of demand for FBCT agencies and test size agencies.

(see §2.2.5). The total demand varied between instances of the same size resulting in the number of depots (K) being (3 or 4), (6 or 7), (12, 13 or 14) for problem instances of sizes 25, 50 and 100, respectively.

3.3.2 Numerical Results

For each of the three problem sizes, the 30 instances were solved exactly¹⁰ using LINGO 11.0. These results were then compared to the CCP-ATS results for the same problem instances. The CCP-ATS was run 10 times for each instance. The average objective function value of the solutions and the best objective function value solution were used in the comparison (see Tables 3.2–3.4).

LINGO 11.0 was able to solve each instance of problem sizes 25 and 50 relatively quickly, with 17 minutes being the longest computation time required to obtain an exact solution. The average solution time for instances of size 25 was 2 seconds, while the average solution time for instances of size 50 was just under 3 minutes. Due to the combinatorial nature of the problem, the number of permutations increases exponentially as the size of the problem increases. This resulted in only 19 instances of size 100 being solved exactly in under 8 hours by LINGO 11.0. For those instances not solved in under 8 hours (marked with an asterisk in Table 3.4), the lower bound on the optimal objective function value at 8 hours was used for comparison purposes. An exact solution for some of these problem instances was still not found when allowed to run for 24 hours. The average solution time (using an 8 hour limit) for instances of size 100 was greater than 4 hours.

These results confirm that as the problem size increases, the average time to find a solution increases dramatically. For the instances of size of 25, only six of the best CCP-ATS solutions were not optimal. For the instances of size 50, none of the best CCP-ATS solutions found were optimal, but the solutions were between 0.1% and 4.32% from the optimal solution found by solving the problem exactly by means of LINGO 11.0. For the instances of size 100, eleven of the instances could not be solved to optimality by LINGO 11.0 within 8 hours. For these 30 instances, none of the best CCP-ATS solutions were optimal, but the solutions were between 1.18% and 5.87% from the optimal solution (or lower bound on the objective) found by solving the problem exactly (for eight hours) by means of LINGO 11.0.

To better understand these results, the percentage difference between the objective function values of the exact solutions and the CCP-ATS solutions (called the *optimality gap*) were fitted

¹⁰For some of the problem instances of size 100, LINGO 11.0 had not found an exact solution after 8 hours. The lower bound on the optimum objective function values after 8 hours was used in the comparison with the CCP-ATS results.

			LINGO 11.0 Solver		Tabu Search Results (10 runs)							
no.	τ	K	Objective		Time		Average Solution			Best Solution		
			Function Value	(s)	Objective Function Value	Time (s)	Percentage Gap	Objective Function Value	Time (s)	Percentage Gap		
1	0.83	4	303.93	3	303.98	1	0.02	303.93	1	0.00		
2	0.83	3	257.08	1	257.08	1	0.00	257.08	1	0.00		
3	0.84	4	341.79	6	349.37	1	2.22	341.78	1	0.00		
4	0.85	3	281.30	1	285.09	1	1.34	281.32	1	0.01		
5	0.85	3	323.09	1	343.04	1	6.18	323.07	1	0.00		
6	0.86	3	274.87	1	275.15	1	0.10	274.88	1	0.00		
7	0.86	3	328.43	1	328.43	1	0.00	328.43	1	0.00		
8	0.86	3	378.35	2	379.55	1	0.32	378.95	1	0.16		
9	0.87	3	347.72	1	350.01	1	0.66	347.70	1	0.00		
10	0.88	3	300.48	1	304.37	1	1.29	300.47	1	0.00		
11	0.88	3	286.56	6	287.24	1	0.24	286.56	1	0.00		
12	0.88	3	329.77	6	329.87	1	0.03	329.76	1	0.00		
13	0.89	3	320.56	1	322.03	1	0.46	320.53	1	0.00		
14	0.89	3	319.31	1	319.31	1	0.00	319.31	1	0.00		
15	0.89	3	334.96	1	346.86	1	3.55	334.96	1	0.00		
16	0.89	3	284.34	1	284.32	1	0.00	284.32	1	0.00		
17	0.90	3	306.97	1	307.38	1	0.13	306.95	1	0.00		
18	0.90	3	317.34	1	328.30	1	3.45	317.34	1	0.00		
19	0.90	3	319.94	1	320.00	1	0.02	319.95	1	0.00		
20	0.91	3	309.23	1	312.00	1	0.89	310.82	1	0.51		
21	0.92	3	286.97	1	287.79	1	0.29	286.99	1	0.01		
22	0.92	3	324.34	2	324.33	1	0.00	324.33	1	0.00		
23	0.92	3	322.25	2	341.45	1	5.96	325.68	1	1.06		
24	0.93	3	349.38	6	350.16	1	0.22	349.38	1	0.00		
25	0.93	3	349.32	1	353.06	1	1.07	352.65	1	0.95		
26	0.94	3	345.56	1	351.13	1	1.61	346.23	1	0.19		
27	0.94	3	321.92	3	322.46	1	0.17	321.93	1	0.00		
28	0.95	3	275.28	1	275.33	1	0.02	275.29	1	0.00		
29	0.95	3	339.07	1	339.57	1	0.15	339.07	1	0.00		
30	0.96	3	335.45	4	336.18	1	0.22	335.45	1	0.00		
Average			2		1		1.02	1		0.10		

Table 3.2: The results of LINGO 11.0 and the CCP-ATS for test problems of size 25.

3.3. Application to Test Problems

47

			LINGO 11.0 Solver		Tabu Search Results (10 runs)										
no.	τ	K	Objective		Time (s)	Average Solution			Best Solution						
			Function Value			Objective Function Value	Time (s)	Percentage Gap	Objective Function Value	Time (s)	Percentage Gap				
1	0.82	7	470.50		3	485.26	1	3.14	482.06	3	2.46				
2	0.84	7	404.37		6	425.15	2	5.14	415.42	2	2.73				
3	0.84	7	472.14		19	480.15	2	1.70	475.23	2	0.65				
4	0.84	7	465.00		5	474.13	2	1.96	466.06	1	0.23				
5	0.84	7	453.50		322	476.94	1	5.17	473.08	2	4.32				
6	0.84	7	473.47		6	483.46	3	2.11	479.48	3	1.27				
7	0.85	7	422.82		16	444.41	2	5.10	438.90	2	3.80				
8	0.85	7	478.28		11	494.44	2	3.38	484.44	4	1.29				
9	0.86	7	442.59		5	461.58	2	4.29	451.32	1	1.97				
10	0.88	7	495.65		717	519.61	2	4.83	508.75	1	2.64				
11	0.88	7	450.44		16	458.52	1	1.79	456.59	2	1.37				
12	0.89	6	478.31		12	488.44	2	2.12	483.52	1	1.09				
13	0.89	7	471.75		19	482.77	2	2.34	475.69	2	0.84				
14	0.89	7	456.45		8	472.45	2	3.51	467.12	3	2.34				
15	0.90	7	471.02		15	482.77	2	2.50	476.35	4	1.13				
16	0.90	7	461.53		38	468.32	1	1.47	464.38	1	0.62				
17	0.91	6	481.10		254	488.70	2	1.58	481.57	2	0.10				
18	0.92	7	445.58		13	458.49	3	2.90	452.61	3	1.58				
19	0.92	6	477.55		161	498.19	2	4.32	482.09	2	0.95				
20	0.93	7	487.52		799	519.08	2	6.47	507.27	2	4.05				
21	0.93	7	503.92		700	523.58	3	3.90	509.69	5	1.14				
22	0.93	6	487.41		8	499.71	1	2.52	493.29	1	1.21				
23	0.94	6	535.59		75	562.62	2	5.05	550.22	3	2.73				
24	0.95	6	455.39		69	467.25	2	2.60	458.45	1	0.67				
25	0.95	6	465.58		12	479.13	2	2.91	467.45	2	0.40				
26	0.95	6	454.58		14	482.94	3	6.24	464.17	4	2.11				
27	0.95	6	493.80		128	516.42	2	4.58	501.20	3	1.50				
28	0.95	6	514.26		975	541.36	2	5.27	532.56	3	3.56				
29	0.95	6	497.80		643	517.93	2	4.04	502.43	1	0.93				
30	0.96	6	479.49		110	521.40	3	8.74	484.51	4	1.05				
Average						2			3.72			2		1.69	

			LINGO 11.0 Solver			Tabu Search Results (10 runs)					
no.	τ	K	Objective		Time (s)	Average Solution			Best Solution		
			Function Value			Objective Function Value	Time (s)	Percentage Gap	Objective Function Value	Time (s)	Percentage Gap
1	0.89	13	690.62		615	735.45	13	6.49	716.33	20	3.72
2	0.89	13	768.50		20 614	806.38	9	4.93	796.24	8	3.61
3	0.89	13	741.48		19 319	772.06	12	4.12	760.67	11	2.59
4	0.89	13	719.29		3 733	756.25	12	5.14	738.93	11	2.73
5	0.89	13	704.14		881	743.29	15	5.56	723.38	15	2.73
6	0.89	14	703.06		2 033	761.51	15	8.31	730.53	17	3.91
7	0.90	13	710.02		4 195	739.51	13	4.15	730.54	19	2.89
8*	0.90	13	715.83		28 800	758.74	13	6.69	745.37	17	4.81
9	0.90	13	689.35		608	731.87	9	6.17	725.17	8	5.20
10	0.90	13	714.24		142	748.29	13	4.77	738.07	16	3.34
11	0.91	13	730.21		1 100	789.90	14	8.17	740.66	15	1.43
12*	0.91	13	736.91		28 800	794.56	14	7.82	780.15	26	5.87
13*	0.92	13	754.43		28 800	789.70	19	4.68	782.30	39	3.69
14	0.92	13	712.70		542	757.02	17	6.22	732.78	17	2.82
15*	0.92	12	750.27		28 800	791.31	16	6.10	772.37	13	3.56
16*	0.92	13	720.37		28 800	754.83	12	4.98	741.70	18	3.16
17	0.92	12	738.93		591	764.94	17	3.52	754.05	33	2.05
18*	0.93	12	759.89		28 800	816.91	15	7.86	803.60	23	6.10
19*	0.93	12	755.55		28 800	781.53	13	3.76	769.15	12	2.11
20*	0.93	13	737.35		28 800	798.38	14	8.63	761.38	16	3.60
21*	0.93	13	726.39		28 800	746.09	11	2.71	737.10	11	1.47
22	0.93	13	757.66		3 454	801.86	16	5.83	793.50	20	4.73
23	0.94	12	719.84		25 664	762.93	17	6.47	748.57	20	4.46
24	0.94	12	750.33		17 376	771.10	13	3.16	764.16	21	2.23
25	0.94	12	709.47		7 874	750.53	11	5.79	736.16	13	3.76
26*	0.95	12	751.20		28 800	805.03	19	7.44	787.06	30	5.04
27	0.95	12	726.65		13 546	756.12	10	4.06	738.16	16	1.58
28*	0.95	12	700.94		28 800	726.26	15	4.01	714.07	13	2.27
29	0.95	12	733.39		12 548	750.02	12	2.50	742.06	14	1.41
30	0.96	12	731.26		14 010	773.67	17	5.80	760.71	17	4.03
Average			15 521			14			5.38		
			18			3.22					

Table 3.4: The results of LINGO 11.0 and the CCP-ATS for test problems of size 100.

	Size 50	Size 100
Kolmogorov-Smirnov		
Sample Size	30	30
Statistic	0.1767	0.0891
P-Value	0.2728	0.9540
α	0.05	0.05
Critical Value	0.2417	0.2417
Reject?	No	No
Anderson-Darling		
Sample Size	30	30
Statistic	0.9853	0.2902
P-Value	—	—
α	0.05	0.05
Critical Value	2.5018	2.5018
Reject?	No	No
Chi-Squared		
Sample Size	30	30
Statistic	4.6745	0.7823
P-Value	0.1973	0.8537
α	0.05	0.05
Critical Value	7.8147	7.8147
Reject?	No	No

Table 3.5: The results of the Kolmogorov-Smirnov, Anderson-Darling and Chi-Squared tests for fitting the results of problem sizes 50 and 100 as normal distributions.

for problem sizes 50 and 100 to a theoretical distribution using EASYFIT [44] software. There where no clear properties of the uncertainty, therefore the choice of distribution is based on the results of the Kolmogorov-Smirnov, Anderson-Darling and Chi-Squared tests [7]. For problem size 100, the normal distribution was ranked very highly. Therefore, the assumption that the data was normally distributed was made and the results of the Kolmogorov-Smirnov, Anderson-Darling and Chi-Squared tests were that this assumption could not be rejected at a 0.05 level of significance. The results of these three tests, calculated using EASYFIT software, can be seen in Table 3.5.

Assuming that the results are normally distributed, the problem size 50 results have a mean of 1.69 and a standard deviation of 1.15 with respect to the optimality gap, while the problem size 100 results have a mean of 3.22 and a standard deviation of 1.27 in this respect (see Tables 3.6 and 3.7). These sample values were used to calculate a 95% confidence interval for the mean of the optimality gap.

For a normal distribution, a $1 - \alpha$ percentage confidence interval for the population mean (μ) from a sample (greater than 30) is given by

$$\bar{X} - z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right), \quad (3.7)$$

where n is the sample size, \bar{X} is the sample mean, s is the standard deviation of the sample and z is the z-score value which represents how many standard deviations a value is from the mean.

For a 95% confidence interval, $z_{\alpha/2} = 1.96$. For the problem instances of size 50, the 30 observations yield the values $\bar{X} = 1.69$ and $s = 1.15$, so that the corresponding confidence

interval is

$$1.69 - 1.96\left(\frac{1.15}{\sqrt{30}}\right) < \mu_{50} < 1.69 + 1.96\left(\frac{1.15}{\sqrt{30}}\right),$$

or, simplified, $1.28 < \mu_{50} < 2.10$. For the problem instances of size 100, the 30 observations yield the values $\bar{X} = 3.22$ and $s = 1.27$, so that the corresponding confidence interval is

$$3.22 - 1.96\left(\frac{1.27}{\sqrt{30}}\right) < \mu_{100} < 3.22 + 1.96\left(\frac{1.27}{\sqrt{30}}\right)$$

or, simplified, $2.77 < \mu_{100} < 3.67$. If it is assumed that the 30 instances are normally distributed and that the population mean and standard deviation are the same as the sample mean and standard deviation for the normal distribution

$$Z = \frac{X - \mu}{\sigma},$$

then it follows from the normal distribution probabilities table that

$$P(Z < 2.33) = 0.99.$$

For the problem size 50, this translates to the inequality

$$\frac{X - 1.69}{1.15} < 2.33$$

or, equivalently to $X < 4.37$, while for the problem size 100 it translates to the inequality

$$\frac{X - 3.22}{1.27} < 2.33$$

or, equivalently $X < 6.18$.

Therefore, under the assumptions made above, 99% of the CCP-ATS objective functions' values of problem instances of size 50 will be within 4.37% of the optimal, while for problem instances of size 100, 99% of the CCP-ATS objective functions' values will be within 6.18% of the optimal.

3.4 Application on FBCT Data and Results

The distance FLP model (3.1)–(3.6) formulated in this chapter was applied to the FBCT data and the results are reported in this section.

3.4.1 Application on FBCT Data

The FBCT FLP requires 158 agencies to be serviced, with a total demand of 83952.5 kilograms (Table 3.8 contains the demand in kilograms of each agency). The model (3.1)–(3.6) was implemented for different numbers of depots, K , ranging from 17 (the smallest feasible value) to 40 (the average number of agencies per depot closest to the value of 4). The CCP-ATS was used to solve the instances where K ranges from 17 to 20, while LINGO 11.0 was used to solve the instances where K ranges from 21 to 40. The model was implemented using the one-way distance between any two points. However, all distances were doubled to obtain the distance of a return trip¹¹. The *warehouse-depot* (WD) distance is the return distance from the warehouse to the depot (the first term in (3.1)) and the *agency-depot* (AD) distance is the return distance from the agency to the depot (the second term in (3.1)).

¹¹The same solution is obtained whether using one-way distances or doubling them to represent a return trip.

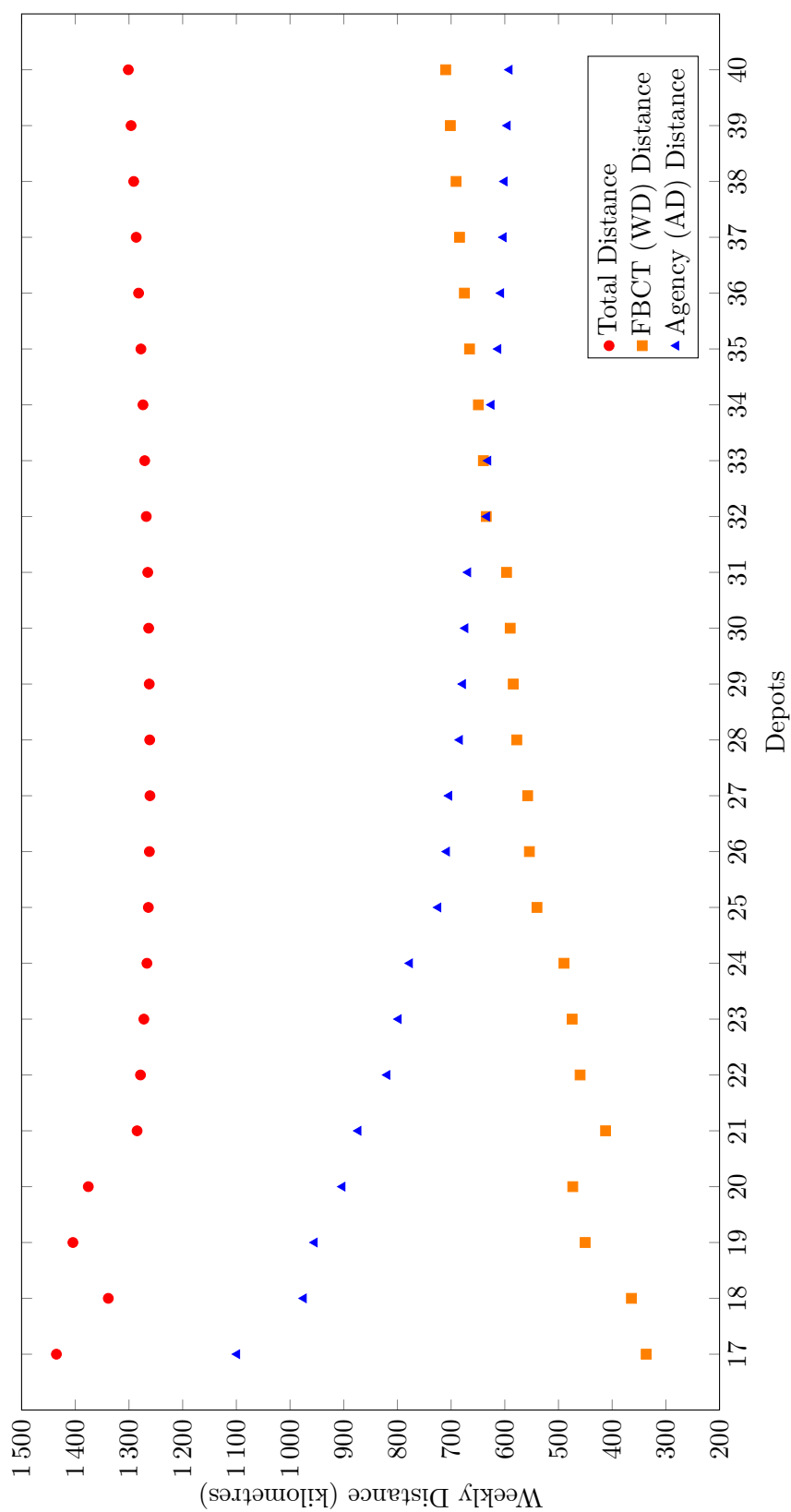


Figure 3.10: The weekly total distance, total WD distance and total AD distance for 17 to 40 depots.

No. (<i>i</i>)	x_i	$x_i - \bar{X}$	$(x_i - \bar{X})^2$
1	2.46	0.77	0.59
2	2.73	1.04	1.08
3	0.65	-1.04	1.08
4	0.23	-1.46	2.13
5	4.32	2.63	6.91
6	1.27	-0.42	0.18
7	3.80	2.11	4.45
8	1.29	-0.40	0.16
9	1.97	0.28	0.08
10	2.64	0.95	0.90
11	1.37	-0.32	0.10
12	1.09	-0.60	0.36
13	0.84	-0.85	0.72
14	2.34	0.65	0.42
15	1.13	-0.56	0.31
16	0.62	-1.07	1.15
17	0.10	-1.59	2.53
18	1.58	-0.11	0.01
19	0.95	-0.74	0.55
20	4.05	2.36	5.56
21	1.14	-0.55	0.30
22	1.21	-0.48	0.23
23	2.73	1.04	1.08
24	0.67	-1.02	1.04
25	0.40	-1.29	1.67
26	2.11	0.42	0.18
27	1.50	-0.19	0.04
28	3.56	1.87	3.49
29	0.93	-0.76	0.58
30	1.05	-0.64	0.41
50.73		38.31	

Table 3.6: Determining the sample mean, $\bar{X} = 1.69$, and standard deviation, $s = 1.15$, of the results for problems of size 50.

No. (<i>i</i>)	x_i	$x_i - \bar{X}$	$(x_i - \bar{X})^2$
1	3.72	0.50	0.25
2	3.61	0.39	0.15
3	2.59	-0.63	0.40
4	2.73	-0.49	0.24
5	2.73	-0.49	0.24
6	3.91	0.69	0.47
7	2.89	-0.33	0.11
8	4.13	0.91	0.83
9	5.2	1.98	3.91
10	3.34	0.12	0.01
11	1.43	-1.79	3.21
12	5.87	2.65	7.01
13	3.69	0.47	0.22
14	2.82	-0.40	0.16
15	2.95	-0.27	0.07
16	2.96	-0.26	0.07
17	2.05	-1.17	1.37
18	5.75	2.53	6.39
19	1.8	-1.42	2.02
20	3.26	0.04	0.00
21	1.47	-1.75	3.07
22	4.73	1.51	2.28
23	3.99	0.77	0.59
24	1.84	-1.38	1.91
25	3.76	0.54	0.29
26	4.77	1.55	2.40
27	1.58	-1.64	2.70
28	1.87	-1.35	1.83
29	1.18	-2.04	4.17
30	4.03	0.81	0.65
96.65		47.03	

Table 3.7: Determining the sample mean, $\bar{X} = 3.22$, and standard deviation, $s = 1.27$, of the results for problems of size 100.

3.4.2 Results

The results of the distance-based FLP for 17 to 40 depots may be seen in Tables 3.11 and 3.9. As the number of depots increases, the total AD distance decreases and the WD distance increases. The lowest total distance was achieved for 27 depots.

The CCP-ATS was used to solve the 17 to 20 depot instances. For 18 depots the total distance was lower than that for 19 or 20 depots; this may be explained by the assumption that a very good solution, close to optimal, was found by means of the CCP-ATS for 18 depots, whereas good solutions (but not as close to optimal) were found for 19 and 20 depots. LINGO 11.0 was used to solve the 20 depot instance; however, a solution could not be found within 12 hours. The lower bound on the objective function value after 12 hours was 1293.84. The CCP-ATS solution for the 20 depots problem is therefore no more than 6.33% from the optimal solution.

The total distance (objective function value), the total WD distance and the total AD distance are plotted in Figure 3.10. It may be seen that for 17 to 31 depots the total WD distance is

Hex	Agency	Demand	Hex	Agency	Demand	Hex	Agency	Demand	Hex	Agency	Demand	Hex	Agency	Demand
1249	0	89.4	1973	32	182.7	2385	64	324.0	2647	96	240.0	3206	128	41.4
1440	1	126.0	2006	33	330.0	2386	65	225.0	2649	97	90.0	3207	129	340.2
1441	2	525.0	2007	34	210.0	2420	66	3780.0	2659	98	600.0	3208	130	300.0
1481	3	30.0	2014	35	510.0	2426	67	861.0	2678	99	180.0	3236	131	321.3
1495	4	302.4	2015	36	189.0	2449	68	1500.0	2688	100	150.0	3256	132	72.0
1533	5	331.2	2019	37	995.4	2453	69	510.0	2707	101	126.0	3268	133	114.0
1549	6	33.0	2060	38	378.0	2462	70	300.0	2708	102	16.8	3307	134	69.3
1596	7	480.0	2066	39	291.0	2480	71	176.4	2743	103	12.0	3312	135	189.0
1599	8	3336.0	2088	40	210.0	2485	72	1449.0	2767	104	60.0	3336	136	546.0
1602	9	723.6	2100	41	450.0	2487	73	495.0	2802	105	90.0	3342	137	60.0
1639	10	441.0	2107	42	60.0	2495	74	126.0	2888	106	318.0	3382	138	759.0
1651	11	315.0	2111	43	3743.7	2520	75	120.0	2905	107	180.0	3405	139	1606.5
1653	12	151.2	2111	44	3743.7	2522	76	291.0	2909	108	450.0	3565	140	31.5
1654	13	651.0	2112	45	345.0	2523	77	168.0	2911	109	16.5	3651	141	630.0
1655	14	75.6	2144	46	300.0	2525	78	205.8	2931	110	129.6	3655	142	1080.0
1743	15	75.0	2156	47	105.0	2529	79	279.0	2936	111	1121.4	3676	143	399.0
1745	16	327.6	2196	48	90.0	2544	80	567.0	2938	112	15.0	3695	144	18.0
1758	17	151.2	2236	49	360.0	2545	81	5.1	2940	113	120.0	3696	145	300.0
1759	18	60.0	2268	50	108.0	2549	82	369.0	2964	114	1500.0	3697	146	428.4
1796	19	600.0	2270	51	1500.0	2552	83	150.0	2968	115	1670.1	3715	147	90.0
1797	20	91.5	2274	52	366.0	2553	84	718.5	2973	116	720.0	3717	148	397.8
1812	21	153.0	2277	53	4788.0	2554	85	699.0	2999	117	126.0	3742	149	120.0
1814	22	207.9	2280	54	88.2	2555	86	417.0	3056	118	180.0	3757	150	90.0
1850	23	81.9	2302	55	357.0	2581	87	378.0	3079	119	176.4	3800	151	15.0
1861	24	930.0	2313	56	960.0	2584	88	138.0	3089	120	90.0	3801	152	409.5
1867	25	1029.0	2344	57	900.0	2585	89	273.0	3119	121	225.0	3802	153	1890.0
1901	26	147.0	2345	58	246.0	2586	90	1044.0	3120	122	9.0	3844	154	75.0
1916	27	27.0	2347	59	252.0	2615	91	651.0	3127	123	249.3	3868	155	300.0
1922	28	264.0	2349	60	159.0	2616	92	1788.0	3136	124	3000.0	3930	156	30.0
1946	29	456.0	2350	61	408.0	2617	93	799.5	3178	125	126.0	3944	157	180.0
1964	30	960.0	2354	62	3.0	2622	94	135.0	3193	126	2141.4			
1970	31	787.5	2355	63	45.0	2642	95	585.9	3205	127	2760.0			

Table 3.8: The list of agency hexagon numbers and their corresponding demands in kilograms.

Depots	Tightness τ	Total dist	Total AD dist	Ave A per D	Ave AD dist	Largest AD dist	AD dist ≥ 20 km	FBCT dist	Ave WD dist	Largest WD dist	Smallest WD dist
17	0.99	1 435.16	1 098.53	9.29	6.95	37.38	7	336.63	19.80	44.68	2.34
18	0.93	1 338.49	974.39	8.78	6.17	40.88	5	364.10	20.23	47.46	2.34
19	0.88	1 404.41	954.11	8.32	6.04	32.84	2	450.31	23.70	57.30	2.34
20	0.84	1 375.73	902.34	7.90	5.71	29.46	3	473.39	23.67	57.30	3.24
21	0.80	1 284.88	872.60	7.52	5.52	37.36	5	412.28	19.63	48.24	2.34
22	0.76	1 278.55	818.81	7.18	5.18	28.34	4	459.73	20.90	48.24	2.34
23	0.73	1 272.26	797.68	6.87	5.05	28.34	4	474.58	20.63	48.24	2.34
24	0.70	1 266.52	776.74	6.58	4.92	28.34	4	489.77	20.41	48.24	2.34
25	0.67	1 263.89	723.82	6.32	4.58	20.46	1	540.06	21.60	54.94	2.34
26	0.65	1 261.98	707.81	6.08	4.48	20.46	1	554.17	21.31	54.94	2.34
27	0.62	1 260.92	703.51	5.85	4.45	20.46	1	557.40	20.64	54.94	2.34
28	0.60	1 261.37	683.69	5.64	4.33	20.46	1	577.67	20.63	54.94	2.34
29	0.58	1 262.22	677.98	5.45	4.29	20.46	1	584.23	20.15	54.94	2.34
30	0.56	1 263.33	673.41	5.27	4.26	20.46	1	589.92	19.66	54.94	2.34
31	0.54	1 265.00	668.27	5.10	4.23	20.46	1	596.74	19.25	54.94	2.34
32	0.52	1 267.76	633.23	4.94	4.01	20.46	1	634.54	19.83	54.94	2.34
33	0.51	1 270.69	630.80	4.79	3.99	20.46	1	639.89	19.39	54.94	2.34
34	0.49	1 273.93	624.76	4.65	3.95	20.46	1	649.17	19.09	54.94	2.34
35	0.48	1 277.74	612.06	4.51	3.87	20.46	1	665.67	19.02	54.94	2.34
36	0.47	1 282.09	606.87	4.39	3.84	20.46	1	675.23	18.76	54.94	2.34
37	0.45	1 286.52	602.24	4.27	3.81	20.46	1	684.28	18.49	54.94	2.34
38	0.44	1 291.18	600.47	4.16	3.80	20.46	1	690.70	18.18	54.94	2.34
39	0.43	1 296.08	594.66	4.05	3.76	20.46	1	701.42	17.99	54.94	2.34
40	0.42	1 301.20	591.12	3.95	3.74	20.46	1	710.08	17.75	54.94	2.34

Table 3.9: The results (part 1) of the FBCT FLP based on distance for 17 to 40 depots.

Depot No.	Hex No.	Agency No.	WD dist	Agencies	Demand	Total A dist	Agencies list
1	1 533	5	35.56	10	2214.60	54.21	0, 3, 5, 10, 15, 16, 19, 20, 23, 26
2	1 596	7	14.11	3	1131.00	10.31	1, 2, 7
3	1 651	11	12.68	4	3980.40	12.80	4, 8, 11, 27
4	1 759	18	20.66	14	4968.60	60.50	6, 9, 12, 13, 14, 17, 18, 21, 22, 25, 28, 32, 37, 39
5	2 014	35	7.56	6	3027.00	17.73	24, 30, 35, 36, 38, 42
6	2 088	40	54.93	2	666.00	4.59	29, 40
7	2 100	41	23.63	5	1647.00	13.83	33, 34, 41, 46, 55
8	2 156	47	15.72	4	4981.20	7.51	31, 44, 45, 47
9	2 345	58	4.88	3	2646.00	8.17	51, 57, 58
10	2 347	59	3.23	1	252.00	—	59
11	2 349	60	9.91	5	2169.00	12.17	49, 52, 56, 60, 64
12	2 350	61	12.22	5	4592.70	15.21	43, 48, 61, 65, 74
13	2 354	62	24.19	4	4924.20	15.36	53, 54, 62, 63
14	2 420	66	2.34	1	3780.00	—	66
15	2 462	70	20.57	5	2190.00	26.15	67, 70, 79, 98, 100
16	2 520	75	13.36	6	2694.00	23.64	50, 72, 75, 91, 96, 101
17	2 554	85	6.42	7	3427.50	14.78	76, 77, 84, 85, 86, 90, 97
18	2 581	87	20.35	6	3014.40	32.04	68, 71, 81, 82, 87, 95
19	2 585	89	10.07	10	4410.30	28.16	69, 73, 83, 88, 89, 92, 93, 99, 102, 104
20	2 743	103	16.43	7	1730.10	47.69	78, 94, 103, 105, 106, 116, 123
21	2 909	108	19.01	6	3812.40	40.35	108, 111, 112, 114, 118, 136
22	2 931	110	32.76	5	4053.00	35.62	80, 107, 110, 119, 124
23	2 940	113	16.46	4	1932.60	6.22	109, 113, 115, 117
24	3 178	125	23.16	13	4607.40	80.06	120, 121, 122, 125, 127, 128, 129, 130, 131, 133, 135, 137, 140
25	3 405	139	41.48	7	4948.20	68.22	126, 132, 134, 138, 139, 149, 157
26	3 696	145	47.45	7	2923.20	24.47	142, 143, 144, 145, 146, 148, 155
27	3 801	152	48.25	8	3229.50	43.72	141, 147, 150, 151, 152, 153, 154, 156
				557.40	158	83952.30	703.51

Table 3.10: The agency-depot assignment for the 27-depot instance of the FBCT FLP.

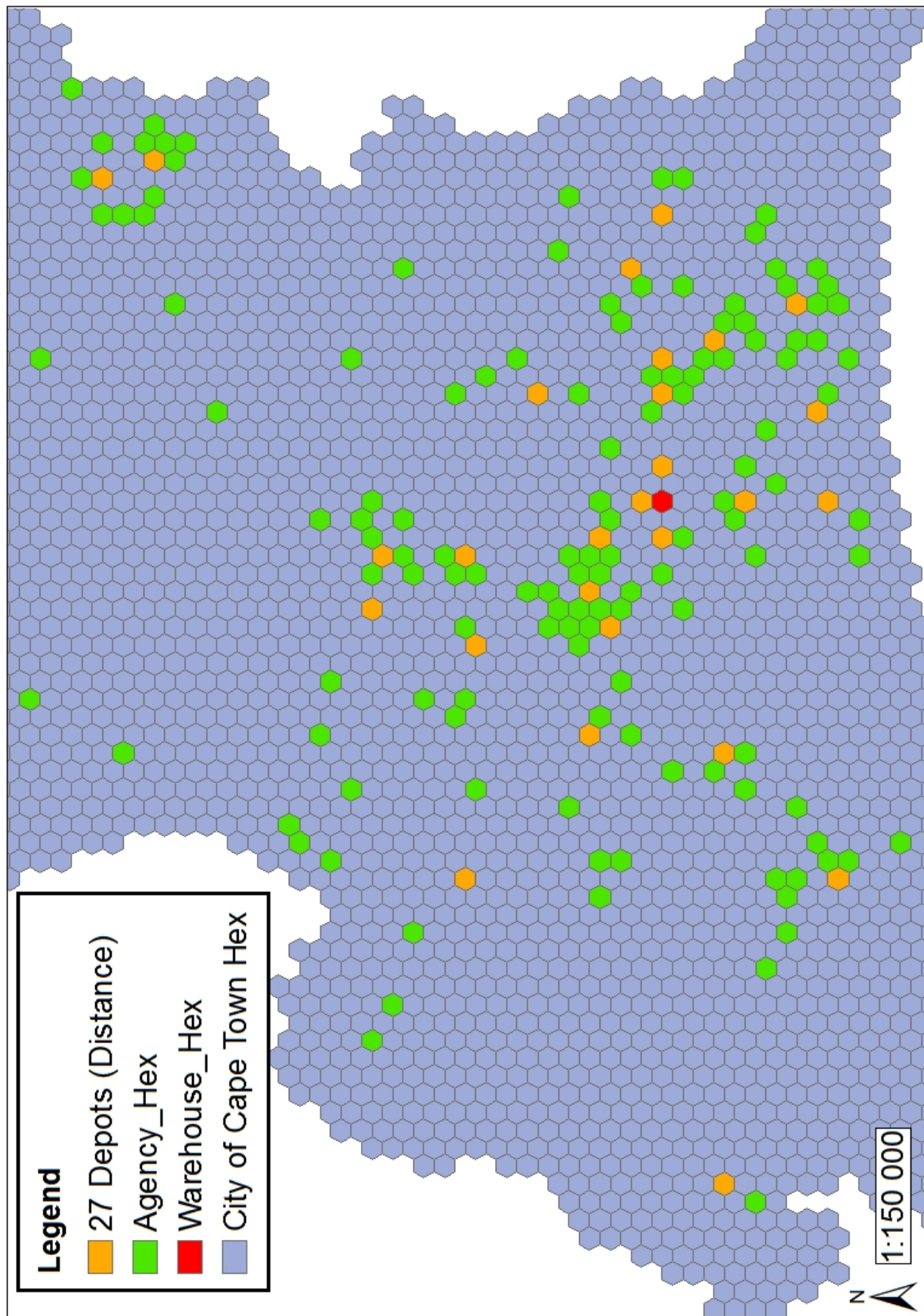


Figure 3.11: The 27 depots selected for the distance-based FLP, mapped in ARCMAP [16].

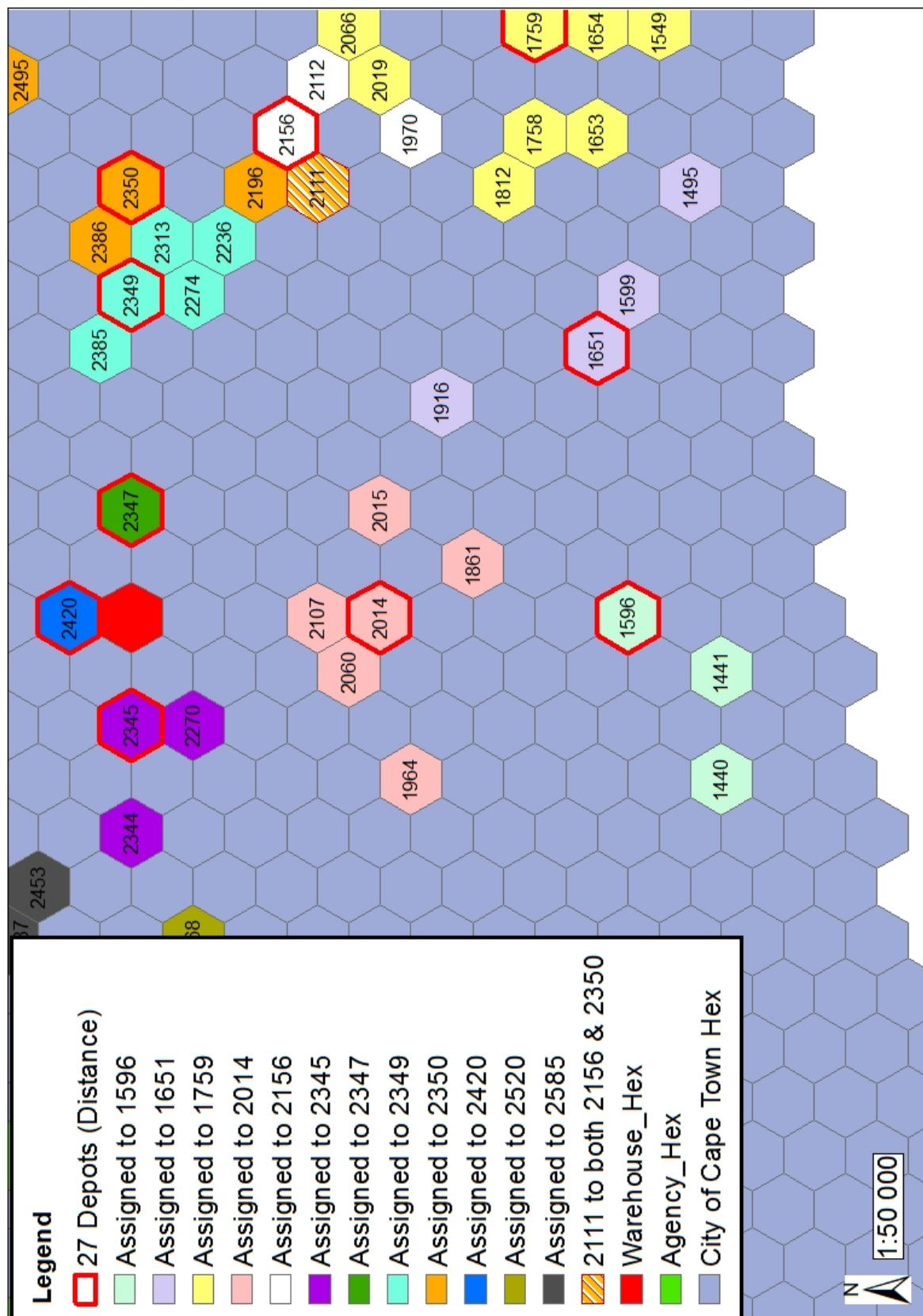


Figure 3.12: A zoomed-in view of the assignment of agencies to depots for the distance-based FLP for 27 depots, mapped in ARCMAP [16].

Depots	Tightness τ	Solution method	Solution time (min)	Total AD distance	Total WD distance	Total distance	FBCT %
17	0.99	CCP-ATS	1	1098.53	336.63	1 435.16	23.46
18	0.93	CCP-ATS	1	974.39	364.10	1 338.49	27.20
19	0.88	CCP-ATS	1	954.11	450.31	1 404.41	32.06
20	0.84	CCP-ATS	1	902.34	473.39	1 375.73	34.41
21	0.80	Exact	73	872.60	412.28	1 284.88	32.09
22	0.76	Exact	55	818.81	459.73	1 278.55	35.96
23	0.73	Exact	27	797.68	474.58	1 272.26	37.30
24	0.70	Exact	27	776.74	489.77	1 266.52	38.67
25	0.67	Exact	49	723.82	540.06	1 263.89	42.73
26	0.65	Exact	53	707.81	554.17	1 261.98	43.91
27	0.62	Exact	18	703.51	557.40	1 260.92	44.21
28	0.60	Exact	23	683.69	577.67	1 261.37	45.80
29	0.58	Exact	234	677.98	584.23	1 262.22	46.29
30	0.56	Exact	237	673.41	589.92	1 263.33	46.70
31	0.54	Exact	237	668.27	596.74	1 265.00	47.17
32	0.52	Exact	33	633.23	634.54	1 267.76	50.05
33	0.51	Exact	13	630.80	639.89	1 270.69	50.36
34	0.49	Exact	25	624.76	649.17	1 273.93	50.96
35	0.48	Exact	7	612.06	665.67	1 277.74	52.10
36	0.47	Exact	38	606.87	675.23	1 282.09	52.67
37	0.45	Exact	10	602.24	684.28	1 286.52	53.19
38	0.44	Exact	4	600.47	690.70	1 291.18	53.49
39	0.43	Exact	4	594.66	701.42	1 296.08	54.12
40	0.42	Exact	6	591.12	710.08	1 301.20	54.57

Table 3.11: The results (part 2) of the FBCT FLP based on distance for 17 to 40 depots.

smaller than the total AD distance, while for 32 to 40 depots the total WD distance is larger than the total AD distance. The large decrease in the total distance from 20 depots to 21 depots is expected due to the 20 depot instance being solved using the CCP-ATS (a good solution whereas the 21 depot instance was solved exactly, thus finding an optimal solution).

An example of the assignment of agencies to depots may be seen in Table 3.10, this table shows the problem solution and agency-depot assignment for the 27 depots instance. The depots selected in the 27 depots instance are shown in Figure 3.11, while Figure 3.12 shows the assignment of agencies to depots for a small selection of agencies for this instance.

3.5 Chapter Summary

The FLP of FBCT based on distance was considered in this chapter. Some background information to the FBCT FLP was given. This was followed by a description of the three different distribution scenarios for the distribution of food from the FBCT warehouse to the agencies. The process of obtaining and processing of data required for the FBCT FLP was explained thereafter; this included GIS data, demand data, infrastructure location, candidate site selection and travel distances.

The distance-based FLP of FBCT was modelled and a number of test instances of this FLP for small problem sizes were generated and solved. A comparison with respect to solution quality and execution time between the CCP-ATS solutions and the exact solutions was performed and a few statistical results were presented on the optimality gap.

The FBCT FPL based on distance was then solved for all numbers of depots from 17 to 40 and the results were presented and analysed. From these results the FLP solution for 27 depots resulted in the smallest total distribution distance. The distance breakdown between agencies and FBCT was provided for 17 to 40 depots.

CHAPTER 4

Facility Location with Vehicle Fleet Composition

Contents

4.1	Model Formulation	61
4.2	Vehicle Cost Calculation	62
4.2.1	FBCT Vehicle Costs	64
4.2.2	Agency Vehicle Costs	65
4.3	Application of Vehicle Routing	65
4.4	Initial Results	69
4.5	Feedback Process and Final Results	71
4.6	Chapter Summary	76

In order to perform an FLP based on cost, the vehicle sizes to be considered and their running costs are required, since the cost of distributing food depends strongly on the *vehicle fleet composition* (VFC) that is performing the distribution. A VFC comparison is performed in this chapter with respect to the cost of distribution of food. Three vehicle capacities are considered in this VFC comparison: vehicles of 5-ton, 2.5-ton and 1.5-ton capacities. All combinations of the three sizes able to perform the distribution, without leaving any vehicle unused, were considered.

4.1 Model Formulation

The FBCT FLP model (3.1)–(3.6) of the previous chapter is adapted slightly in this section. An FLP based on *agency-depot* (AD) cost and *warehouse-depot* (WD) cost is formulated as a mixed-integer linear programming problem, with the objective of minimising the total cost,

$$g = \sum_{j=1}^m c_{0j}x_j + \sum_{i=1}^n \sum_{j=1}^m c_{ij}y_{ij}, \quad (4.1)$$

of food collection by each agency from its assigned depot and of food distribution from the warehouse to each depot. Here c_{0j} denotes the cost of distribution from the warehouse to depot j and c_{ij} denotes the cost of distribution to agency i from the depot j , while

$$y_{ij} = \begin{cases} 1 & \text{if the agency } i \text{ is assigned to depot } j \\ 0 & \text{otherwise} \end{cases}$$

and

$$x_j = \begin{cases} 1 & \text{if depot is located at vertex } j \\ 0 & \text{otherwise.} \end{cases}$$

are binary decision variables. The minimisation of g in (4.1) should be carried out subject to the constraints

$$\sum_{j \in J} x_j = K, \quad j = 1, \dots, m \quad (4.2)$$

$$\sum_{j \in J} y_{ij} = 1, \quad i = 1, \dots, n \quad (4.3)$$

$$y_{ij} \leq x_j, \quad \begin{matrix} i = 1, \dots, n \\ j = 1, \dots, m \end{matrix} \quad (4.4)$$

$$\sum_{i \in I} h_i y_{ij} \leq C_j x_j, \quad j = 1, \dots, m \quad (4.5)$$

$$x_j, y_{ij} \in \{1, 0\}, \quad \begin{matrix} i = 1, \dots, n \\ j = 1, \dots, m \end{matrix} \quad (4.6)$$

where K is the number of depots, h_i is the demand of agency i and C_j is the capacity of depot j . Here the constraint set (4.2) ensures that exactly K facilities are located, while the constraint sets (4.3) and (4.6) ensure that the demand of an agency is assigned to one depot. Constraint set (4.4) ensures that demand is only assigned to a depot which has actually been located and constraint set (4.5) ensures that the demand assigned to a depot does not exceed the capacity of the depot.

The part of the distribution cost incurred by FBCT is the cost of distributing food from the warehouse to each of the depots, that is $\sum_{j=1}^m c_{0j} x_j$, while the part of the distribution cost incurred by the agencies is the cost from each agency to its assigned depot, that is $\sum_{i=1}^n \sum_{j=1}^m c_{ij} y_{ij}$.

The objective function may be rewritten as

$$g = \sum_{j=1}^m b \mu d_{0j} x_j + \sum_{i=1}^n \sum_{j=1}^m a_i d_{ij} y_{ij}, \quad (4.7)$$

where b is the operating cost per kilometre for FBCT vehicles, d_{0j} is the distance from the warehouse to depot j , μ is a distance correction factor (a fraction that represents the distance travelled when vehicle routing is applied to the distribution divided by the distance if no vehicle routing is applied to the distribution), a_i is the operating cost per kilometre for the vehicle of agency i and d_{ij} is the distance between agency i and depot j . If vehicle routing is applied to the distribution, there may be savings on distance travelled as delivery trips are merged, resulting in a value of μ smaller than unity, while if smaller vehicles have to perform multiple trips to a depot, then additional travel distance is incurred, resulting in a value of μ greater than unity.

4.2 Vehicle Cost Calculation

During initial meetings in 2010, FBCT had six vehicles available for collection and delivery of food; four of the vehicles had a 1.5-ton capacity and two had a 2.5-ton capacity. However,

during 2011 FBCT owned a vehicle fleet consisting of five collection and delivery vehicles and additional support motor vehicles. Four of these collection and delivery vehicles had a 1.5-ton capacity and one had a 5-ton capacity. A 5-ton capacity vehicle was considered the largest available option to perform delivery of food to depots, due to the nature of the roads and space required to deliver food to the agencies (which were candidate depot locations). Therefore, the three vehicle sizes decided upon for the VFC comparison in this chapter were 1.5-ton, 2.5-ton and 5-ton vehicles.

In order to determine the cost of food distribution, the vehicle operating costs of the three sizes of vehicles had to be determined. The vehicle costs were calculated with the help of Janse van Rensburg [36]. These calculations were based on the *Road Freight Association's* (RFA's) *Vehicle Cost Schedule* (VCS) [49]. An overview of fixed costs per annum for the three vehicle sizes may be seen in Table 4.1, while the variable costs per kilometre for the three vehicle sizes are shown in Table 4.2.

Fixed Costs	1.5-ton Rand/annum	2.5-ton Rand/annum	5-ton Rand/annum
Cost of Capital (Finance)	10 613	14 160	16 519
Depreciation	33 701	44 065	50 759
Insurance	17 688	23 600	27 532
On Vehicle Staff	54 000	54 000	54 000
Overheads Administration	15 678	30 012	36 283
Overheads Operational	10 452	20 008	24 189
Licence	324	708	852
Total Fixed Cost	142 455	186 553	210 135

Table 4.1: Fixed costs of vehicles of different load capacities [36, 49].

The cost price for the vehicles specified by the RFA VCS were R235 835, R314 668 and R367 097 for the 1.5, 2.5 and 5-ton vehicles, respectively. The annual *cost of capital* in Table 4.1 is half of the cost price of the vehicle multiplied by the interest rate. The interest rate used was 9%, as specified by the RFA VCS for April 2011. Annual *depreciation* was determined as the cost price less the cost of tyres and the residual value, divided by the depreciation years. The depreciation years was taken as 5 years and the residual value as 25% of cost price, as specified by the RFA VCS for April 2011. Annual *insurance* was taken as a percentage of cost price; 7.5% was used in this respect, as specified by the RFA VCS for April 2011. The annual *on vehicle staff* cost was taken as the monthly salary that FBCT pays its drivers [10]. The annual *administration overheads*, *operational overheads* and *licence fees* were obtained from the RFA VCS for April 2011.

Variable Costs	1.5-ton cent/kilometre	2.5-ton cent/kilometre	5-ton cent/kilometre
Fuel	106.3	144.9	212.5
Lubricants	2.7	3.6	5.3
Maintenance	60.6	94.2	123.9
Tyres	13.0	19.7	26.7
Total Variable Cost	182.5	262.5	368.4

Table 4.2: Variable costs of vehicles of different load capacities [36, 49].

The *fuel* cost per kilometre in Table 4.2 is the fuel consumption (litres per 100 kilometres) multiplied by the cost per litre of fuel. The fuel consumption was specified by the RFA VCS for

April 2011 as 11, 15 and 22 litres per 100 kilometres for 1.5, 2.5 and 5-ton vehicles, respectively. The cost per litre was taken as the fuel price per litre in April 2011 for 95 octane unleaded fuel at the coast, namely R9.66, as specified by the *Auto-mobile Association* (AA) [2]. The *lubricants* cost per kilometre was taken as 2.5% of the *fuel* cost; this percentage was specified by the RFA VCS for April 2011. The *maintenance* cost for each vehicle size was also specified by the RFA VCS for April 2011 and the *tyre* cost was calculated based on values of new tyres or retreading costs and the life of a new and retreaded tyre provided by RFA VCS.

The variable costs may be converted to an annual cost if the annual kilometres are known, by multiplying the variable cost per kilometre by the annual kilometres. Alternatively, the annual cost may also be converted to a cost per kilometre value, if the annual kilometres are known, by dividing the fixed annual cost by the annual kilometres.

The FBCT vehicles used for the warehouse-to-depot distribution, service this distribution exclusively. Therefore, the annual kilometres and annual fixed costs are known. Each agency, however, performs its own individual trip to collect food from its assigned depot, while the rest of the vehicle's daily or weekly usage is unknown. The fixed cost per annum of an agency vehicle is therefore not the AD cost for an agency, as the agency vehicle would typically be used for many different purposes besides the collection of food from the depot. Since it is necessary to account for the cost of a trip to the depot from an agency and the only known factor is the distance from the agency to the depot, the fixed cost per annum must be converted to a per kilometre cost. Since the agency cost c_{ij} is based on a cost per kilometre a_i , the FBCT cost c_{0j} also has to be based on a cost per kilometre b .

4.2.1 FBCT Vehicle Costs

For an FBCT vehicle, the fixed cost per week and variable cost per kilometre may be used to calculate the total WD distribution cost, as shown in Table 4.3. However, in the cost model (4.2)–(4.7) a cost per kilometre value b is required. The cost per kilometre value may be calculated using the annual kilometres for each vehicle. When multiple vehicle types are used, the cost per kilometre for each vehicle should be calculated separately and then b may be calculated by taking a weighted average, namely the vehicle type's cost per kilometre multiplied by the fraction of the total distance that the vehicle type travels.

Fixed Costs	1.5-ton	2.5-ton	5-ton
Fixed Cost (Rand per annum)	142 455	186 553	210 135
Fixed Cost (Rand per week)	2 739	3 587	4 041
Variable Cost (cent per kilometre)	182.5	262.5	368.4

Table 4.3: FBCT vehicle costs for three different vehicle load capacities as calculated from Tables 4.1 and 4.2.

Example 4.2.1.1 Suppose the FBCT vehicle fleet consists of a 5-ton vehicle and a 1.5-ton vehicle. The return distance from the warehouse to all depots is 597 kilometres. Suppose the vehicles are routed and that the 5-ton vehicle travels 337 kilometres in a week, while the 1.5-ton vehicle travels 282 kilometres a week. The annual distance of the 5-ton vehicle rounded to nearest 100 kilometres is therefore 17 500 kilometres and the annual distance of the 1.5-ton vehicle rounded to nearest 100 kilometres is therefore 14 700 kilometres. The 5-ton vehicle performs 54% of the total travel distance at a cost of R15.69 per kilometre and so its weekly cost is R5 282. The 1.5-ton vehicle performs 46% of the total travel distance at a cost of R11.52 per

kilometre and consequently the weekly cost is R3 255. The weighted average cost per kilometre b is therefore R13.79 per kilometre and the distance correction factor μ is 103.66% (the additional distance incurred as a result of multiple trips by the 1.5-ton vehicle is larger than the distance saved by vehicle routing applied to the deliveries). ■

4.2.2 Agency Vehicle Costs

For the agency vehicles, the annual kilometres were based on Road Traffic and Fatal Crash Statistics for 2003 and 2004 [50]. The average annual kilometres travelled for all vehicle types in South Africa in 2003 was 18 614 kilometres, while in 2004 it was 18 627 kilometres. This represents a growth of 0.0007%. This growth rate was extrapolated from the 2004 average annual kilometres for each vehicle type for seven years to obtain average annual kilometres for each vehicle type in 2011. This resulted in 16 967 average annual kilometres for a motorcar in 2011 and 21 037 average annual kilometres for a *bakkie/light delivery vehicle* (LDV). These values were rounded to the nearest 100 kilometres, resulting in 17 000 kilometres and 21 000 kilometres for motorcars and LDVs, respectively.

The assumption was made that a collection load smaller than 300 kilograms would be made by a motorcar and that a collection load larger than 300 kilograms would require a LDV. Of the agencies with a weekly demand smaller than 1 500 kilograms, approximately 50% of the agencies exhibited a demand less than 300 kilograms. Therefore, for agencies with a weekly demand smaller than 1 500 kilograms, the annual vehicle kilometres were based on 50% motorcars (17 000 annual kilometres) and 50% LDV (21 000 annual kilometres), resulting in an average of 19 000 annual kilometres for such an agency. For agencies with a weekly demand larger than 1 500 kilograms, only LDVs would be used, resulting in 21 000 annual kilometres for such agencies. These assumptions facilitated an estimation of the total cost per kilometre for the three different vehicle sizes, based on the fixed cost per annum and variable cost per kilometre, as shown in Table 4.4. The cost per kilometre value a_i for agency i was determined by the weekly demand of the agency as one of three values: 932.3 cents per kilometre for a weekly demand smaller than 1 500 kilograms, 1 150.8 cents per kilometre for a weekly demand smaller than 2 500 kilograms and larger than 1 500 kilograms, and 1 369.1 cents per kilometre for a weekly demand larger than 2 500 kilograms.

Agency Vehicle	1.5-ton	2.5-ton	5-ton
Fixed Cost (Rand per annum)	142 455	186 553	210 135
Annual kilometres	19 000	21 000	21 000
Fixed Cost (cent per kilometre)	749.8	888.3	1 000.7
Variable Cost (cent per kilometre)	182.5	262.5	368.4
Total Cost (cent per kilometre)	932.3	1 150.8	1 369.1

Table 4.4: Agency vehicle costs for three different vehicle load capacities, as calculated from Tables 4.1 and 4.2.

4.3 Application of Vehicle Routing

Once the facility location model had been implemented and the depots assigned to agencies, the demand for all agencies assigned to each depot were consolidated to yield a distribution demand

for each depot. The warehouse was included as depot 0. A depot-to-depot distance matrix was generated using the distances calculated previously between all agencies.

The Clarke-Wright algorithm [6] was used to generate the delivery trips that are required to satisfy the depot demand. If a vehicle capacity was smaller than the depot demand, a trip was generated for every full vehicle load required to satisfy the demand. Note that these trips service the depots exclusively. The remaining demand was included in the setup of the Clarke-Wright algorithm, with each depot serviced individually as a single return trip from the warehouse to the depot as an initial solution. The algorithm is applied, merging trips if possible, and returning an approximately optimal set of trips. The full vehicle load trips were added together to give a full set of trips that are required to satisfy the delivery demand.

A total time

$$TT_i = LT_i + OT_i + DT_i$$

was associated with trip i , where LT_i is the load time associated with trip i , OT_i is the offload time associated with trip i and DT_i is the travel (drive) time associated with trip i . These values were calculated as

$$LT_i = \frac{DQ_i}{LS} + LA,$$

$$OT_i = \frac{DQ_i}{OS} + OA \times KS_i,$$

$$DT_i = \frac{DD_i}{DS},$$

where DQ_i is the delivery demand associated with trip i , KS_i is the number of depots delivered to during trip i , DD_i is the travel (drive) distance associated with trip i , LS is the load speed, LA is the load administration time, OS is the offload speed, OA is the offload administration time and DS is the travel (drive) speed.

The load time LT_i was calculated using a fixed load administration time LA of 20 minutes and a load speed LS of 0.008 minutes per kilogram (125 kilograms per minute); this resulted a full load taking 60, 40 and 32 minutes for a 5, 2.5 and a 1.5-ton vehicle, respectively, to be loaded. The offload time OT_i was calculated using a fixed offload administration time OA of 10 minutes and an offload speed OS of 0.004 minutes per kilogram (250 kilograms per minute); this resulted a full load taking 30, 20 and 16 minutes for a 5, 2.5 and a 1.5-ton vehicle, respectively, to be offloaded. These times were based on details from FBCT of a 5-ton vehicle taking approximately an hour to load and a 1.5-ton vehicle taking approximately half an hour to load, and that an offload can be performed in half the time of a load as the food for the delivery is just removed from the vehicle whereas loading needs to be planned so as to accommodate the order of deliveries to be made.

A travel speed DS of 19.9 kilometres per hour was used for all vehicles. This value was calculated using 47 817 kilometres travelled by four 1.5-ton vehicles in 2 635 travel hours over a five-month period from January to May 2011 (see Table 4.5), which gave rise to an average travel time of 0.3025 kilometres per minute (3.31 minutes per kilometre). Different speeds were considered for different vehicle sizes. However, due to the average speed being low in comparison to the capabilities of the vehicles, the speed of a vehicle was found to be more tightly constrained by other traffic than by the size of the vehicle.

Vehicle Month	Dyna 2		Dyna 3		Dyna 4		Dyna 6		Total		Ave
	km	hours	km	hours	km	hours	km	hours	km	hours	
January	1 212	54.9	2 604	150.2	3 002	83.5	3 699	227.9	10 517	516.5	20.4
February	1 520	51.4	2 609	185.1	3 378	97.2	2 495	198.7	10 002	532.4	18.8
March	1 457	39.7	3 078	162.0	3 472	93.0	3 430	229.2	11 437	523.9	21.8
April	1 075	33.5	1 875	117.0	2 884	77.3	3 587	229.9	9 421	457.6	20.6
May	1 627	65.9	2 817	223.3	3 184	84.3	3 512	230.7	11 140	604.2	18.4
Total									52 517	2 634.7	19.9

Table 4.5: The travel distances and travel times for four 1.5-ton FBCT vehicles over the period January to May 2011.

Once the Clarke-Wright algorithm had been applied, the trips thus generated were arranged in non-increasing order of total time, TT_i . The trips were then assigned to delivery days (see Algorithm 4.1). A trip was assigned to the first delivery day which could accommodate it. Delivery days were restricted to 8 hours. Five delivery days were assumed per week for each available vehicle. The result was a five-day schedule for each vehicle. Note that the delivery days are interchangeable, that the order of daily trips are interchangeable and that a trip may also be performed in reverse order. An example of a trip assignment to delivery days may be seen in Table 4.6; this is the routing for the distribution to depots of the solution of the distance-based FLP for 27 depots with a VFC of one 5-ton and one 1.5-ton vehicle. For vehicle sizes of 1.5 and 2.5-tons, it is possible for a delivery to a depot to be split over more than one day. This would, however, require some additional planning regarding which day each agency assigned to the depot should collect its assigned food. Moreover, if an agency's food is split over different delivery days, then planning is required so that the food delivered on an earlier day would still be in good condition when the agency collects this food.

Algorithm 4.1: Trip Assignment

Data: Set of trips with completion time, days available for delivery

Result: Assignment of trips to days

- 1 Set $t \leftarrow$ total trips, $d \leftarrow$ available delivery days and $w \leftarrow$ daily limit;
 - 2 Arrange trips in increasing order of time;
 - 3 Set $i \leftarrow 1$;
 - 4 **while** $i \leq t$ **do**
 - 5 **if** $TT_i + D_j < w$ **then**
 - 6 Trip i assigned to day j ;
 - 7 $D_j \leftarrow D_j + TT_i$;
 - 8 $i++$;
 - 9 **else**
 - 10 $j++$;
 - 11 **if** $j > d$ **then**
 - 12 Trip i is unassigned;
 - 13 $i++$;
 - 14 **Print** trip assignment;
-

If a fleet composition comprised two different sizes, the larger vehicles size were routed first. The trips not assigned after the larger vehicles had been routed were then routed using the smaller vehicles. The reason for this convention is that the longer trips (usually longer travel distances)

Size	Day/Trip	Route	Demand	Kilometres	Time (min)	Time (hour)
5-ton	Day 1					
	Trip 1	0, 7, 1, 6, 0	4 527.60	61.57	289.96	4.83
	Trip 2	0, 23, 18, 0	4 947.00	27.44	182.10	3.03
5-ton	Day 2					
	Trip 1	0, 20, 26, 10, 0	4 905.30	49.86	259.19	4.32
	Trip 2	0, 25, 0	4 948.20	41.48	214.43	3.57
5-ton	Day 3					
	Trip 1	0, 27, 0	3 229.50	48.24	214.20	3.57
	Trip 2	0, 22, 0	4 053.00	32.76	177.41	2.96
	Trip 3	0, 14, 0	3 780.00	2.34	82.42	1.37
5-ton	Day 4					
	Trip 1	0, 13, 0	4 924.20	24.20	162.05	2.70
	Trip 2	0, 15, 11, 0	4 359.00	21.41	156.86	2.61
	Trip 3	0, 24, 0	4 607.40	23.16	155.12	2.59
5-ton	Day 5					
	Trip 1	0, 4, 0	4 968.60	20.66	151.91	2.53
	Trip 2	0, 8, 0	4 981.20	15.72	137.17	2.29
	Trip 3	0, 21, 0	3 812.40	19.00	133.03	2.22
5-ton	Total		58 043.40	387.84	2 315.85	38.60
1.5-ton	Day 1					
	Trip 1	0, 12, 3, 0	1 073.10	19.50	111.67	1.86
	Trip 2	0, 5, 2, 0	1 158.00	14.27	96.92	1.62
	Trip 3	0, 16, 0	1 500.00	13.36	88.28	1.47
	Trip 4	0, 3, 0	1 500.00	12.68	86.23	1.44
1.5-ton	Day 2					
	Trip 1	0, 12, 0	1 500.00	12.22	84.84	1.41
	Trip 2	0, 12, 0	1 500.00	12.22	84.84	1.41
	Trip 3	0, 12, 0	1 500.00	12.22	84.84	1.41
	Trip 4	0, 16, 0	1 194.00	13.36	84.61	1.41
1.5-ton	Day 3					
	Trip 1	0, 19, 0	1 500.00	10.08	78.39	1.31
	Trip 2	0, 19, 0	1 410.30	10.08	77.31	1.29
	Trip 3	0, 5, 0	1 500.00	7.56	70.79	1.18
	Trip 4	0, 5, 0	1 500.00	7.56	70.79	1.18
1.5-ton	Day 4					
	Trip 1	0, 17, 0	1 500.00	6.42	67.36	1.12
	Trip 2	0, 17, 0	1 500.00	6.42	67.36	1.12
	Trip 3	0, 17, 0	1 500.00	6.42	67.36	1.12
	Trip 4	0, 17, 0	1 500.00	6.42	67.36	1.12
1.5-ton	Day 5					
	Trip 1	0, 9, 0	1 500.00	4.88	62.71	1.05
	Trip 2	0, 17, 0	427.50	6.42	54.49	0.91
	Trip 3	0, 17, 0	1 500.00	6.42	67.36	1.12
	Trip 4	0, 17, 0	1 500.00	6.42	67.36	1.12
1.5-ton	Total		25 908.90	196.89	1 494.53	24.91
All	Total		83 952.30	584.73	3 810.38	63.51

Table 4.6: The vehicle routing of one 5-ton and one 1.5-ton vehicle for the distance-based FLP with 27 depots.

are typically performed by the vehicle size with the greatest capacity, while the depots serviced by smaller vehicle sizes are closer to the warehouse, resulting in less (repeated) distance travelled to depots when multiple delivery trips are required.

Due to the nature of the FLP instances considered in this thesis only one or two vehicle sizes were used for each instance. For each instance, if a 5-ton and 2.5-ton vehicle were routed, all demand was satisfied and a third 1.5-ton vehicle was superfluous. Similarly, for each instance, when a 5-ton vehicle was routed first, only two vehicles were required to perform the distribution. However, if a 5-ton vehicle was not in the fleet composition, then three vehicles were required for some instances.

4.4 Initial Results

In order to obtain initial values for the FBCT cost per kilometre b and the distance correction factor μ , vehicle routing was applied to the solutions of the distance-based FLP instances in Chapter 3. The vehicle routing was applied for each FBCT VFC and number of depots K combination (from 17 to 40 depots). The vehicle routing provided a weekly kilometres value for each vehicle and a total weekly kilometres value for each VFC. The initial value for μ was calculated for the given VFC as the total weekly kilometres divided by the total return distance between the warehouse and the depots if no vehicle routing was applied. Weekly kilometres values were converted to annual kilometres values, rounded to the nearest 100 kilometres. The annual kilometres was used to calculate the initial total FBCT vehicle cost per kilometre b , using the annual fixed cost and variable cost per kilometre. For a given number of depots K , the solution to the distance-based FLP is the same, no matter which FBCT VFC is adopted. The AD cost would therefore be the same and the difference in cost would be the difference in the FBCT WD cost, regardless of the VFC. An initial comparison of VFC could then be carried out on the results of this initial routing for each VFC by multiplying the initial cost per kilometre b by the initial total kilometres travelled. The results of the initial VFC comparison are shown in Figure 4.1.

In the initial VFC results, the total WD cost for each VFC tends to increase slightly as the number of depots increases; this is to be expected as more depots results in larger distances travelled and therefore higher costs of distribution. The total WD cost for each VFC decreases from 20 depots to 21 depots, but this can be explained by noting that the distance-based FLP for 20 depots was solved by means of the CCP-ATS and the resulting solution is good but not optimal, while the distance-based FLP for 21 depots was solved exactly by means of LINGO 11.0 (and hence the solution is optimal).

The VFC comprising one 5-ton vehicle and one 1.5-ton vehicle appears to be the best VFC for all values of K between 17 and 40 in these initial results, with the second best VFC comprising one 5-ton vehicle and one 2.5-ton vehicle for 17, 18 and 31 to 40 depots, or alternatively a VFC comprising two 2.5-ton vehicles for 19 to 30 depots.

From these results it may be seen that for the VFC of two 2.5-ton vehicles there is a critical point at 30 depots; this VFC is not sufficient to satisfy demand in the distance-based FLP solution for $K > 30$, which requires a composition of three 2.5-ton vehicles, or two 2.5-ton vehicles and one 1.5-ton vehicle to satisfy the demand.

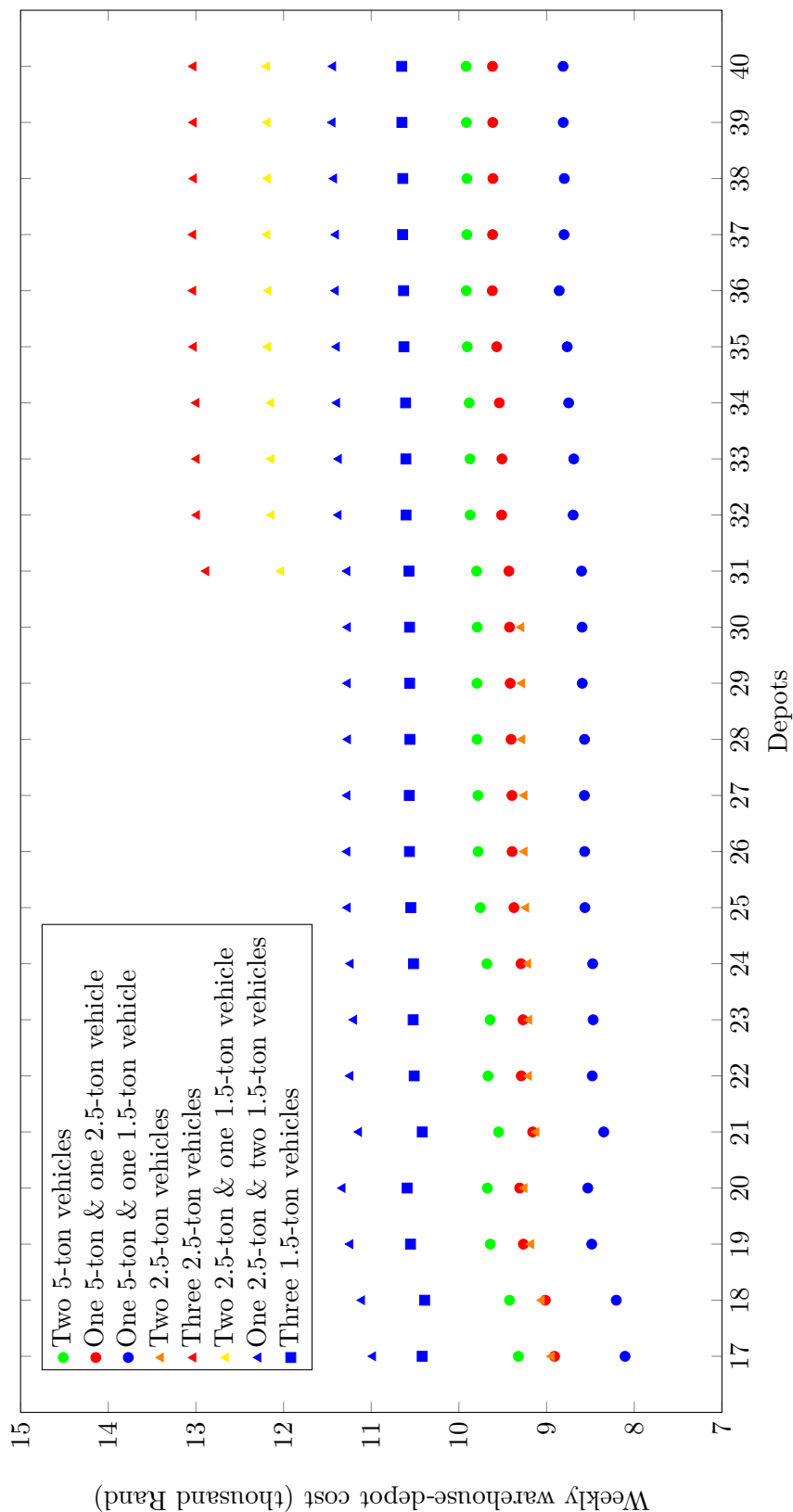


Figure 4.1: The initial results for the total weekly WD cost for 17 to 40 depots.

4.5 Feedback Process and Final Results

Once the initial results had been obtained, the cost-based FLP model (4.2)–(4.7) could be implemented with the values μ and b obtained from the initial results for each VFC and number of depots combination. Once the solution for the cost-based FLP had been found, vehicle routing was applied to this new solution. Due to the average WD cost per kilometre being higher than the average AD cost per kilometre, the cost-based FLP solution results in a smaller total return WD distance and a larger total AD distance. Furthermore, the depots assigned are closer to the warehouse, on average, resulting in a decreased distance for the FBCT vehicles when routed. The new routing with new smaller weekly kilometres value for FBCT vehicles results in the new annual kilometres value being lower too. However, as the annual kilometres value decreases, the total cost per kilometre of a vehicle increases. The new solution and vehicle routing results in a new μ -value and a new, larger b -value, due to the lower annual kilometres value.

The cost-based FLP was solved again using these new values for μ and b , and the routing procedure was again applied. This entire process was repeated until the solution converged, where the cost-based FLP solution and vehicle routing returns the same solution as during the previous iteration. Due to this feedback solution process being extremely time consuming, taking as many as five iterations for convergence to occur, it was not applied for all VFCs and number of depots combinations. It was decided to rather focus on solving the 18 (CCP-ATS), 25, 30, 35 and 40 (LINGO 11.0) depot instances with the four cheapest VFCs found during the initial result (the VFCs comprising two 5-ton vehicles, one 5-ton and one 2.5-ton vehicle, one 5-ton and one 1.5-ton vehicle, and two 2.5-ton vehicles). The results of the total cost for these four VFCs may be seen in Figure 4.2.

For 35 and 40 depots (where the value of $K > 30$), the VFC comprising two 2.5-ton vehicles is able to satisfy the demand of all depots for the cost-based FLP unlike in the distance-based FLP in the previous chapter. This is due to the cost-based FLP producing a solution in which the total WD distance is less than the total WD distance of the distance-based FLP solution. There do not appear to be any critical points for the four VFCs applied to the cost-based FLP.

It was clear that the VFC comprising one 5-ton and one 1.5-ton vehicle was the best combination for 18, 25, 30, 35 or 40 depots and the assumption was made that this would also be true for other numbers of depots in the range 17 to 40. The VFC comprising one 5-ton and one 1.5-ton vehicle was therefore used to solve the cost-based FLP for all numbers of depots in the range 17 to 40. The results in terms of the total cost and distance travelled may be seen in Figures 4.3 and 4.4.

As the number of depots increases for the VFC comprising one 5-ton and one 1.5-ton vehicle, the total cost of distribution decreases, the agencies' cost decreases and the FBCT cost increases slightly. For 17 to 29 depots the agencies' cost is larger than the FBCT cost, for 30 depots the FBCT cost and agencies' cost is very close to equal, and for 31 to 40 depots the FBCT cost is larger than the agencies' cost.

For the VFC comprising one 5-ton and one 1.5-ton vehicle, the total weekly distance remains relatively constant for 21 to 40 depots, solved exactly by means of LINGO 11.0. However as the number of depots increases the FBCT travel distance tends to increase and the agencies' travel distance tends to decrease. The cost-based FLP solution results in a smaller WD distance than the WD distance for the distance-based FLP solution. An example of this may be seen in Figure 4.5, showing the difference in depots selected for the two solutions with 30 depots, where, in general, the depots in the cost-based FLP solution tend to be closer to the warehouse.

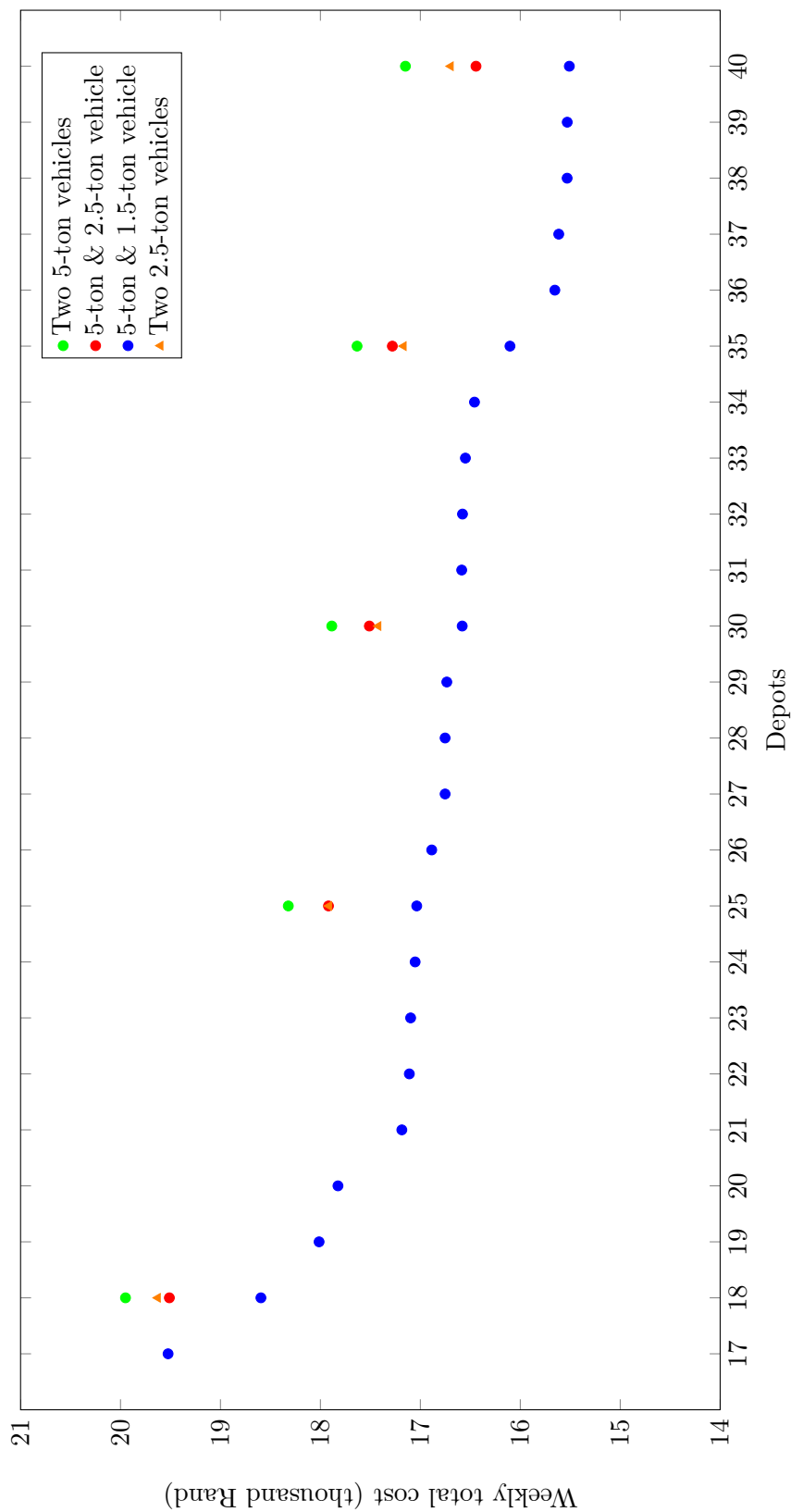


Figure 4.2: The results for the total cost for four VFCs for 18, 25, 30, 35 and 40 depots and the VFC comprising one 5-ton and one 1.5-ton vehicle for 17 to 40 depots.

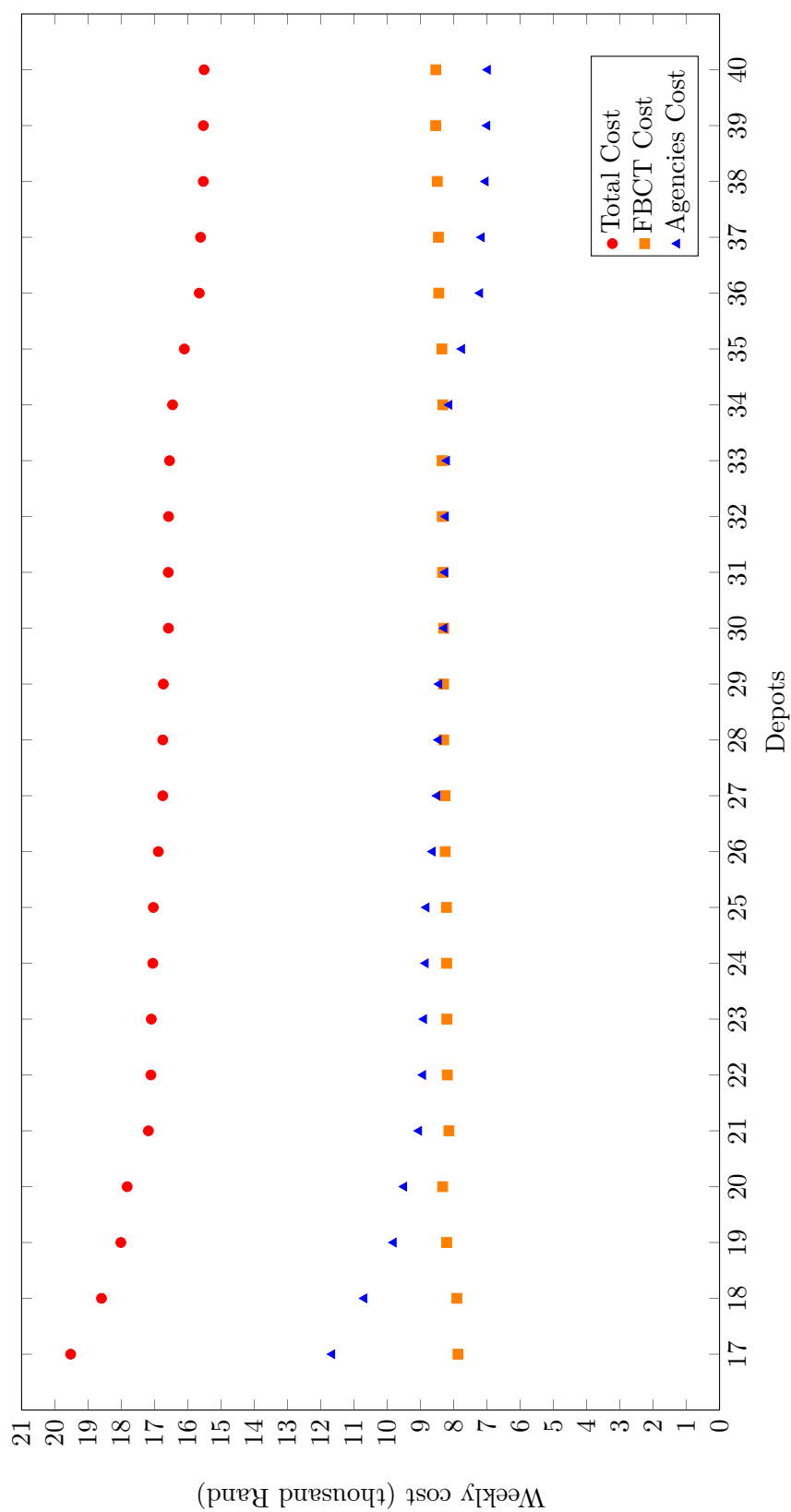


Figure 4.3: Breakdown of the total cost for the VFC of one 5-ton and one 1.5-ton vehicle for 17 to 40 depots.

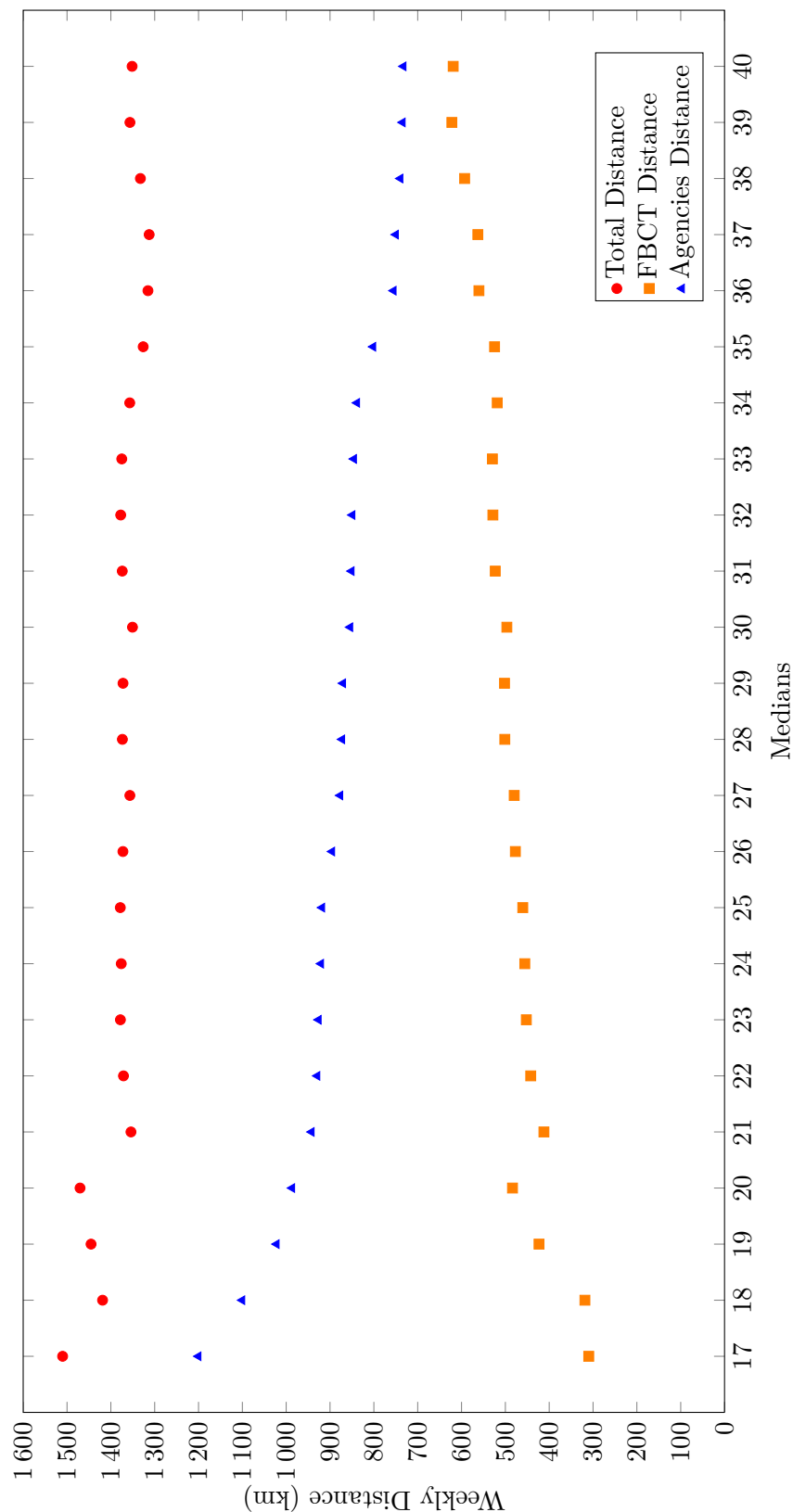


Figure 4.4: Breakdown of the total distance for the VFC of one 5-ton and one 1.5-ton vehicle for 17 to 40 depots.

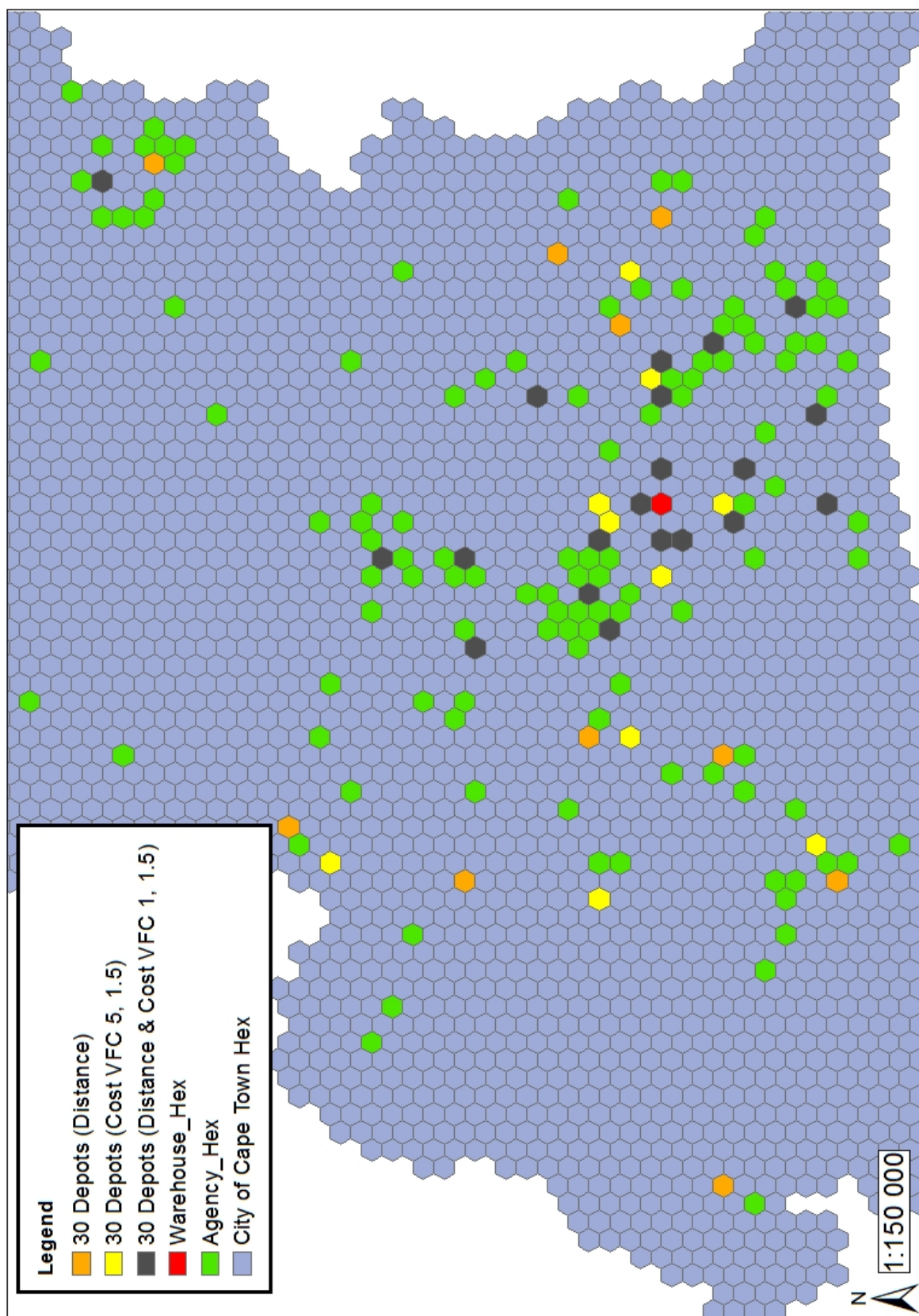


Figure 4.5: Comparison of depots selected in the distance-based and cost-based FLPs for the 30-depot instance, mapped in ARCMAP [16]. The two solutions have 20 depots in common.

For all numbers of depots (from 17 to 40) the agencies' distance is larger than the FBCT distance. The agencies' percentage of total distance is also larger than the agencies' percentage of cost for the same number of depots; this may be attributed to the average agency cost per kilometre being smaller than the average FBCT cost per kilometre — therefore, a larger agencies' distance than FBCT distance is incurred for the same cost.

4.6 Chapter Summary

The FBCT FLP based on cost was considered in this chapter. The cost-based FLP was modelled and the vehicle costs were calculated, based on data provided by the RFA VCS. The application of vehicle routing to FBCT vehicles was then considered. Initial values for the distance correction factor μ and the FBCT cost per kilometre parameter b were obtained by routing the distance-based FLP according to a given VFC. The feedback process for updating the distance correction factor μ and the FBCT cost per kilometre parameter b , by repeatedly solving the cost-based FLP until the solution converged, was explained.

The four cheapest VFCs from the initial results (two 5-ton vehicles, one 5-ton and one 2.5-ton vehicle, one 5-ton and one 1.5-ton vehicle, and two 2.5-ton vehicles) were used to solve the cost-based FLP for 18, 25, 30, 35 and 40 depots. From these results it was found that the VFC comprising one 5-ton and one 1.5-ton vehicle resulted in the lowest total cost in the cost-based FLP. This VFC was then used to solve all cost-based FLP instances for 17 to 40 depots and a total cost and distance breakdown between agencies and FBCT was provided.

CHAPTER 5

Conclusion

Contents

5.1 Thesis Summary	77
5.2 Recommendations to FBCT	78
5.3 Feedback from FBCT	79
5.4 Challenges of this Study	79
5.5 Future Work	80

This concluding chapter opens with a summary of the work contained in this thesis. This is followed by a number of recommendations to FBCT regarding the facility location and VFC and by feedback received from FBCT on the work presented. The chapter closes with a discussion on ideas for future work that may be done in collaboration with FBCT and FBSA.

5.1 Thesis Summary

The concept of foodbanking was introduced in Chapter 1, including how and why it started and its development into a global system working actively in the fight against hunger. An overview of FBSA and FBCT was given before the scope of this thesis was presented. Four FBCT focus areas, namely food allocation, distribution logistics, activity-based costing and future foodbank locations, were discussed and the selection of distribution logistics as the focus for this thesis was motivated. The objectives for the work in this thesis were set out and the organisation of the work to follow was described.

A review of models in the operations research literature related to the FLP were presented in Chapter 2, in fulfilment of Objective *I* (see §1.3). The chapter also contains a review of appropriate solution techniques to these models and the software used in this thesis.

The FLP of FBCT was outlined at the start of Chapter 3, giving some background information to the FBCT problem. This was followed by a description of three different distribution scenarios for the distribution of food from the FBCT warehouse to agencies it serves. Next the details of how data were obtained and processed were explained, including GIS data, demand data, infrastructure location, candidate site selection and travel distances, in fulfilment of Objective *II* parts (a) and (b), and in partial fulfilment of Objective *II* part (c). The FLP for FBCT based on distance was formulated next, in fulfilment of Objective *III* part (a). This was followed by the generation and solution of small test instances of the FLP; this included the application of

the CCP-ATS meta-heuristic as well as an exact solution method to these test instances. A comparison was carried out between the CCP-ATS solutions and the exact solution in terms of solution quality and execution time, in fulfilment of Objective *III* parts (b) and (c). The FBCT FPL based on distance was then solved for all numbers of depots (from 17 to 40) and the results were presented, in fulfilment of Objective *V* part (a). The more highly constrained problem instances (17 to 20 depots) were solved using the CCP-ATS meta-heuristic, while the less constrained problem instances were solved exactly using LINGO 11.0, in fulfilment of Objective *III* part (d). A breakdown of the distances travelled by FBCT vehicles and by agency vehicles are also presented, in partial fulfilment of Objective *V* part (c).

Chapter 4 opened with the formulation of the FLP for FBCT based on cost, in partial fulfilment of Objective *III* part (a). This was followed by an overview of vehicle costs assumed in the model, considering both FBCT vehicle costs and agency vehicle costs, in fulfilment of Objective *II* part (c). The application of a basic vehicle routing technique to FBCT vehicles followed next. Initial results were obtained and an initial VFC was analysed by applying the vehicle routing technique to the solutions of the distance-based FLP. This was done to obtain an initial distance correction factor and initial FBCT cost per kilometre that could be used to solve the cost-based FLP, in partial fulfilment of Objective *IV* part (b). The cost-based FLP was solved next and the feedback process required to obtain convergence was explained, in fulfilment of Objective *IV* part (a). The four VFCs with the lowest associated initial FBCT vehicle costs were used to solve the cost-based FLP for 18, 25, 30, 35 and 40 depots, in fulfilment of Objective *IV* part (b). For the best VFC, the cost-based FLP was solved for all numbers of depots (17 to 40) and the results were presented together with a breakdown of both cost of distribution and distance travelled by FBCT vehicles and agency vehicles, in partial fulfilment of Objective *V* parts (a) and (c).

5.2 Recommendations to FBCT

One of the most important reasons why FBCT wanted to establish local distribution depots was so that the cost incurred or distance travelled in respect of the distribution of food could be shared between itself and the agencies it serves. Although obtaining a lowest cost or distance for the entire distribution may seem the best solution, an important aspect of selecting a preferred number of depots was the breakdown of the cost incurred or distance travelled, with a view to have the distance travelled or cost incurred shared as evenly as possible between FBCT and its agencies.

When the distance-based FLP was solved the minimum number of depots possible was 17, this resulted in the largest total distance (1 435 kilometres); FBCT would, however, travel under a quarter of the total distance. The solution to the 27-depots instance achieved the lowest total distance (1 261 kilometres) with FBCT travelling approximately 44% of the total distance. The solution achieving the most evenly shared distance was found to be with 32 depots, with almost an equal share of distance travelled by FBCT and its agencies (only a total distance of 8 kilometres more than that of the 27 depots instance solution).

When the cost-based FLP was solved, the VFC comprising one 5-ton and one 1.5-ton vehicle appeared to be the best VFC for the FBCT vehicles. For this VFC, the cost for 17 (the minimum number of) depots of R19 524 was the largest total cost, with FBCT incurring just over 40% of this cost. The solution to the 40-depots instance achieved the lowest total cost (R15 509) with FBCT incurring 55% of the cost. Since the 40-depots instance contained the largest number of depots, it may be assumed that an instance with more depots would result in an even lower total

cost; this solution, however, had an average of just under four agencies per depot and FBCT had suggested that an average of between 5 and 10 agencies per depot would be preferable. The solution with the most evenly spread cost was found to be the one with 30 depots, with an almost equal share of the total cost, but with a cost of approximately R1 070 more than in the 40-depots instance solution. The cost to FBCT is approximately R240 cheaper in this case, but the cost to the agencies being over R1 300 more expensive. However, the average cost increase for each agency equates to just over R8 a week in this case. Therefore the number 30 seems a good choice for the number of depots that FBCT should establish. It also achieves an average of just over 5.25 agencies per depot, which is within the range of agencies per depots that FBCT initially preferred. The solution to the cost-based FLP instance with 30 depots may be seen in §A.2.

5.3 Feedback from FBCT

The level of importance of establishing local distribution depots is not as high as it had been during 2010. The nature of FBCT distribution has changed somewhat since then and due to financial constraints their focus has shifted to more pressing issues. FBCT have established a few local distribution depots at agencies with whom they have very good relationships, while deliveries are made to some agencies and some agencies still collect from the FBCT warehouse.

At the moment FBCT is considering where it can expand its operations in serving needy communities better. This has resulted in the focus being shifted from decision support to costing of its operations and these expanded operations, so that they can allocate or raise the funds to cover the cost of these operations.

The results obtained in this thesis in terms of the routing of vehicles and cost breakdown between FBCT and the agencies were of interest to FBCT. A more specialised focus on comparing different costing options would, however, be of most benefit to FBCT. This would focus on comparing two different agencies requiring delivery, in order to determine which would be most suitable to be added to the current deliveries. The cost to agencies is often overlooked and therefore an idea of what costs agencies incur in comparison to FBCT was seen as very valuable during the feedback session with FBCT.

Futhermore, FBSA is currently in the process of evaluating the possibility of creating a separate small business enterprise focussed on logistics and contracting this business to perform collection and distribution for FBCT. The vehicle costs and vehicle routing aspects were seen to be tools which could be very helpful during decisions about creating a separate small business enterprise.

5.4 Challenges of this Study

When work first started on this thesis, FBCT had been operating for just under a year. As a new organisation it faced many challenges with many structures still to be put in place. Significant changes to the organisation have occurred during the course of work towards this thesis; these changes and new developments have brought both FBSA and FBCT a long way since the initial meetings in the beginning of 2010 outlined in the Terms of Reference.

Part of these changes and developments occurred within the personnel structure of FBCT and FBSA, as certain roles became more crucial to the operations of both FBSA and FBCT. These changes resulted in working with different people to those who proposed project ideas and a

focus for research in early 2010. Also, the perceived level of importance of certain operations areas changed during this time; especially the move to establish more foodbanks became a more pressing issue. Feeding more people in different areas of the country promotes FBSA's brand, giving the organisation more exposure, which is vital to fulfilling its role as a vehicle for generating greater public awareness and involvement in the fight to end hunger.

Obtaining data was a difficult task at times. Identifying the right person to contact for the required data was at times unclear, especially with the above-mentioned changes in personnel. Being a relatively new organisation, certain crucial data were not always recorded at the level of detail that would have been preferred.

These changes and difficulties could not have been foreseen when the scope of this thesis was being thrashed out.

5.5 Future Work

This section contains three suggestions for possible future work with respect to the FLP of establishing local distribution depots for FBCT and other related problems for FBSA. Since the scope of any research project is restricted, there is always a possibility for improvement.

Solving the FBCT FLP for a new warehouse

Solving the FBCT FLP for the location of a new FBCT warehouse is an avenue of investigation that may be pursued, again applying vehicle routing techniques, performing a VFC comparison and applying similar methods and processes to the work done in this thesis with new data in the form of a new warehouse location and different agencies. A similar process may also be followed for the other cities in which FBSA has established a presence, allowing each foodbank to improve its distribution logistics operations.

Building a decision support tool

A computerised decision support tool may be designed that incorporates the FBCT FLP, vehicle routing and VFC components of this thesis. Such a tool would have to be user-friendly and the different aspects of vehicle routing and VFC should be accessible in isolation, allowing FBCT to apply these aspects to collection operations, should the collection of food become more consistent and stabilised in the future. The tool may also be integrated with GIS software to facilitate a better visual representation of results.

Investigating the effect of larger depot capacities

An investigation into the effect of using a smaller number of depots with larger capacities that can accommodate multiple deliveries of food during a week is another topic for possible future research. This idea stemmed from the results of routing smaller (1.5 and 2.5-ton) vehicles. This would require FBCT to build stronger relationships with the more reliable agencies that would be prepared to have other agencies collect food from them on multiple days of the week.

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APPENDIX A

Additional Tables and Results

Contents

A.1 5-ton and 1.5-ton VFC Results	85
A.2 30-depot Cost-based FLP Solution	85

This appendix contains additional results (in table form) not included in full in the main body of the thesis.

A.1 5-ton and 1.5-ton VFC Results

Tables A.1 and A.2 show the results of the FBCT FLP based on cost for 17 to 40 depots with a VFC comprising one 5-ton and one 1.5-ton vehicle.

A.2 30-depot Cost-based FLP Solution

The solution for the 30-depot cost-based FBCT FLP with a VFC comprising one 5-ton and one 1.5-ton vehicle is shown in Tables A.3 and A.4. Table A.3 contains the depot selection and the agency assignment to depots, while Table A.4 contains the vehicle routings to service the 30 depots.

Depots	Tightness τ	Total cost	Total AD cost	Total WD cost	FBC T % of cost	Total distance	Total AD distance	Total WD distance	FBC T % of distance
17	0.99	19 524	11 655	7 869	40.30	1 510	1 200	310	20.52
18	0.93	18 595	10 690	7 905	42.51	1 419	1 100	319	22.45
19	0.88	18 013	9 805	8 209	45.57	1 445	1 022	423	29.28
20	0.84	17 824	9 491	8 332	46.75	1 470	987	484	32.90
21	0.80	17 185	9 042	8 143	47.38	1 354	942	412	30.43
22	0.76	17 110	8 921	8 190	47.87	1 371	929	442	32.24
23	0.73	17 097	8 893	8 204	47.99	1 378	926	452	32.81
24	0.70	17 052	8 842	8 211	48.15	1 376	921	456	33.11
25	0.67	17 036	8 819	8 217	48.23	1 378	918	460	33.39
26	0.65	16 886	8 632	8 254	48.88	1 372	895	477	34.77
27	0.62	16 752	8 494	8 258	49.30	1 357	877	480	35.37
28	0.60	16 752	8 453	8 299	49.54	1 374	872	501	36.49
29	0.58	16 735	8 434	8 302	49.61	1 372	870	502	36.58
30	0.56	16 581	8 282	8 299	50.05	1 351	854	497	36.77
31	0.54	16 586	8 253	8 333	50.24	1 374	851	523	38.07
32	0.52	16 578	8 235	8 343	50.33	1 378	849	528	38.37
33	0.51	16 549	8 202	8 347	50.44	1 375	845	530	38.52
34	0.49	16 458	8 132	8 326	50.59	1 357	838	519	38.21
35	0.48	16 103	7 748	8 354	51.88	1 326	802	525	39.56
36	0.47	15 654	7 206	8 449	53.97	1 315	755	560	42.60
37	0.45	15 615	7 157	8 458	54.17	1 313	750	563	42.88
38	0.44	15 531	7 039	8 492	54.68	1 333	740	593	44.50
39	0.43	15 530	6 990	8 540	54.99	1 356	734	622	45.86
40	0.42	15 509	6 972	8 536	55.04	1 352	732	619	45.80

Table A.1: The results (part 1) of the FBC T FLP based on cost (one 5-ton and one 1.5-ton VFC) for 17 to 40 depots.

Depots	Total cost	Total AD cost	Ave A per D	Ave AD cost	Largest AD cost	Kg value of largest AD cost	Largest A cost per kg	Kg value of largest cost per kg	WD cost	FBCT % of cost	Ave WD cost	Utilisation % of 5-ton vehicle	Utilisation of % 1.5-ton vehicle
17	19 524	11 655	9.29	73.77	431.97	3 000.0	39.04	3.0	7 869	40.30	462.88	91.12	17.69
18	18 595	10 690	8.78	67.66	429.91	1 890.0	24.98	3.0	7 905	42.51	439.17	95.20	13.49
19	18 013	9 805	8.32	62.06	322.40	1 080.0	24.98	3.0	8 209	45.57	432.05	98.67	28.55
20	17 824	9 491	7.90	60.07	322.40	1 080.0	24.98	3.0	8 332	46.75	416.60	95.98	42.60
21	17 185	9 042	7.52	57.23	429.91	1 890.0	24.98	3.0	8 143	47.38	387.76	94.38	33.94
22	17 110	8 921	7.18	56.46	429.91	1 890.0	24.98	3.0	8 190	47.87	372.27	94.62	37.48
23	17 097	8 893	6.87	56.28	429.91	1 890.0	24.98	3.0	8 204	47.99	356.70	94.66	39.54
24	17 052	8 842	6.58	55.96	429.91	1 890.0	24.98	3.0	8 211	48.15	342.13	94.66	40.81
25	17 036	8 819	6.32	55.82	429.91	1 890.0	24.12	3.0	8 217	48.23	328.68	91.78	46.36
26	16 886	8 632	6.08	54.63	429.91	1 890.0	24.12	3.0	8 254	48.88	317.46	92.05	49.88
27	16 752	8 494	5.85	53.76	429.91	1 890.0	24.12	3.0	8 258	49.30	305.85	92.35	49.48
28	16 752	8 453	5.64	53.50	429.91	1 890.0	24.12	3.0	8 299	49.54	296.39	92.47	53.31
29	16 735	8 434	5.45	53.38	429.91	1 890.0	24.12	3.0	8 302	49.61	286.28	92.98	53.31
30	16 581	8 282	5.27	52.42	429.91	1 890.0	24.12	3.0	8 299	50.05	276.63	93.73	52.30
31	16 586	8 253	5.10	52.23	429.91	1 890.0	24.12	3.0	8 333	50.24	268.81	93.49	56.29
32	16 578	8 235	4.94	52.12	429.91	1 890.0	24.12	3.0	8 343	50.33	260.72	93.35	58.34
33	16 549	8 202	4.79	51.91	429.91	1 890.0	24.12	3.0	8 347	50.44	252.94	93.92	58.34
34	16 458	8 132	4.65	51.47	429.91	1 890.0	24.12	3.0	8 326	50.59	244.88	94.37	55.66
35	16 103	7 748	4.51	49.04	429.91	1 890.0	24.12	3.0	8 354	51.88	238.69	95.48	56.99
36	15 654	7 206	4.39	45.61	308.45	456.0	24.12	3.0	8 449	53.97	234.69	93.20	65.83
37	15 615	7 157	4.27	45.30	308.45	456.0	24.12	3.0	8 458	54.17	228.59	93.92	65.83
38	15 531	7 039	4.16	44.55	308.45	456.0	24.12	3.0	8 492	54.68	223.47	93.66	69.89
39	15 530	6 990	4.05	44.24	308.45	456.0	24.12	3.0	8 540	54.99	218.97	93.82	75.47
40	15 509	6 972	3.95	44.13	308.45	456.0	24.12	3.0	8 536	55.04	213.40	93.61	75.71

Table A.2: The results (part 2) of the FBCT FLP based on cost (one 5-ton and one 1.5-ton VFC) for 17 to 40 depots.

Depot No.	Hex No.	Agency No.	Agencies	Demand	Total A cost	Agencies list
1	1596	7	3	1 131.00	96	1, 2 7
2	1639	10	11	2607.60	590	0, 3, 5, 10, 15, 16, 19, 20, 23, 33, 34
3	1651	11	3	3953.40	85	4, 8, 11
4	1759	18	14	4968.60	564	6, 9, 12, 13, 14, 17, 18, 21, 22, 25, 28, 32, 37, 39
5	2015	36	3	1146.00	69	24, 27, 36
6	2060	38	3	1848.00	66	30, 35, 38
7	2107	42	1	60.00	-	42
8	2156	47	4	4981.20	80	31, 44, 45, 47
9	2270	51	1	1500.00	-	51
10	2344	57	2	1410.00	36	57, 69
11	2345	58	1	246.00	-	58
12	2347	59	1	252.00	-	59
13	2349	60	5	2169.00	113	49, 52, 56, 60, 64
14	2350	61	5	4592.70	165	43, 48, 61, 65, 74
15	2386	65	1	4788.00	103	53
16	2420	66	1	3780.00	-	66
17	2449	68	8	4119.90	469	41, 46, 55, 68, 82, 87, 95, 107
18	2462	70	8	2326.20	493	54, 62, 63, 67, 70, 79, 98, 100
19	2520	75	6	2694.00	220	50, 72, 75, 91, 96, 101
20	2523	77	1	168.00	-	77
21	2544	80	9	4867.50	1234	26, 29, 40, 71, 80, 81, 110, 119, 124
22	2554	85	4	2124.00	63	76, 85, 90, 97
23	2555	86	1	417.00	-	86
24	2585	89	10	4618.80	247	73, 83, 84, 88, 89, 92, 93, 99, 102, 104
25	2743	103	8	3620.10	875	78, 94, 103, 105, 106, 116, 123, 153
26	2909	108	6	3812.40	376	108, 111, 112, 114, 118, 136
27	2940	113	4	1932.60	64	109, 113, 115, 117
28	3178	125	13	4607.40	767	120, 121, 122, 125, 127, 128, 129, 130, 131, 133, 135, 137, 140
29	3307	134	7	4948.20	717	126, 132, 134, 138, 139, 149, 157
30	3801	152	14	4262.70	789	141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 154, 155, 156
			158	83952.30	8282	

Table A.3: The agency-depot assignment for the 30-depot instance.

Size	Day/Trip	Route	Demand	Kilometres	Time (min)	Time (hour)
5-ton	Day 1					
	Trip 1	0, 30, 0	4 262.70	48.24	226.60	3.78
	Trip 2	0, 29, 0	4 948.20	38.88	206.60	3.44
5-ton	Day 2					
	Trip 1	0, 11, 10, 2, 0	4 263.60	34.69	205.75	3.43
	Trip 2	0, 21, 0	4 867.50	31.40	183.08	3.05
5-ton	Day 3					
	Trip 1	0, 27, 19, 0	4 626.60	22.12	162.21	2.70
	Trip 2	0, 18, 13, 0	4 495.20	21.41	158.49	2.64
5-ton	Day 4					
	Trip 1	0, 28, 0	4 607.40	23.16	155.12	2.59
	Trip 2	0, 4, 0	4 968.60	20.66	151.91	2.53
5-ton	Day 5					
	Trip 1	0, 25, 12, 0	3 872.10	17.03	137.81	2.30
	Trip 2	0, 8, 0	4 981.20	15.72	137.17	2.29
5-ton	Day 6					
	Trip 1	0, 17, 0	4 119.90	18.82	136.18	2.27
5-ton	Total		62 207.40	329.33	2 249.42	37.49
1.5-ton	Day 1					
	Trip 1	0, 15, 14, 3, 0	1 334.10	19.56	124.98	2.08
	Trip 2	0, 22, 24, 20, 0	910.80	10.65	93.04	1.55
	Trip 3	0, 3, 0	1 500.00	12.68	86.23	1.44
	Trip 4	0, 3, 0	1 500.00	12.68	86.23	1.44
1.5-ton	Day 2					
	Trip 1	0, 14, 0	1 500.00	12.22	84.84	1.41
	Trip 2	0, 14, 0	1 500.00	12.22	84.84	1.41
	Trip 3	0, 15, 0	1 500.00	10.90	80.86	1.35
	Trip 4	0, 15, 0	1 500.00	10.90	80.86	1.35
1.5-ton	Day 3					
	Trip 1	0, 24, 0	1 500.00	10.08	78.39	1.31
	Trip 2	0, 24, 0	1 500.00	10.08	78.39	1.31
	Trip 3	0, 24, 0	1 500.00	10.08	78.39	1.31
	Trip 4	0, 9, 0	1 500.00	5.68	65.13	1.09
1.5-ton	Day 4	—	—	—	—	—
1.5-ton	Day 5	—	—	—	—	—
1.5-ton	Total		21 744.90	167.27	1 255.26	20.92
All	Total		83 952.30	496.60	3 504.68	58.41

Table A.4: The vehicle routing of one 5-ton and one 1.5-ton vehicle for the cost-based FLP with 30 depots.

APPENDIX B

Contents of the accompanying compact disk

This appendix contains a brief description of the compact disc included with this thesis. The compact disk contains an electronic version of the thesis itself in pdf format, the Java code associated with the CCP-ATS (used to solve the highly constrained instances of the FBCT FLP model), the Java code associated with the Clarke-Wright algorithm and the assignment of delivery days (used to perform the routing of vehicles), GIS data, problem instance data and the full solutions of both the distance-based and cost-based FLP instances.

The compact disc contains the following six directories:

Thesis This directory contains an electronic copy of this thesis in pdf format.

Code This directory contains each piece of implementation code associated with the Clarke-Wright algorithm and the CCP-ATS as well as the input files for these pieces of code.

GIS This directory contains two folders, Original and Created. The folder Original contains all the GIS shape files that were received from various sources and the folder Created contains the GIS shape files that were created during work towards this thesis.

Data This directory contains two folders, Original and Created. The folder Original contains all the excel spreadsheet files that were received from FBCT during 2010–2011 and the folder Created contains the excel spreadsheet files that were created in processing data.

Distance-based FLP Results This directory contains the solutions to instances of the FLP based on distance.

Cost-based FLP Results This directory contains four folders with the solutions to the FLP based on cost. Each folder contains the solutions to one of the four different VFCs used to solve the cost-based FLP.